Auctions of Real Options*

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Abstract

Governments and corporations frequently auction assets with embedded real options using both cash and contingent bids. I characterize equilibrium bidding and option exercise strategies, and find that the moral hazard associated with the uncontractible investment timing inefficiently and asymmetrically accelerates or delays investments. I use a mechanism design approach instead of security "steepness" to rank securities and derive the optimal security. Furthermore, without sellers' commitment to the security design, all auction equilibria are equivalent to cash auctions, and investments are socially efficient. The results are broadly consistent with empirical observations, for example in the sales of oil leases.

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1 Introduction

On March 30, 2013, the Bureau of Ocean Energy Management held an oil lease auction that netted the U.S. government \$1.2 billion. Exxon-Mobil emerged as a dominant bidder and was entitled, but not obliged, to develop seven of the 320 auctioned tracts in the Central Gulf of Mexico, and was to pay the government 18.75% royalty on future oil production.[1] Less than two years later, Shire PLC acquired NPS Pharmaceutical, Inc. for approximately \$5.2 billion in cash to grow NPS's portfolio of licenses and products through its global footprint, expertise in gastrointestinal disorders, and capabilities in rare disease patient management.[2] These two deals involve classic examples of real options whose sale and exercise underlie some of the most crucial decisions for entrepreneurs, firm executives, and government officials. These transactions also routinely see competing bids in combinations of cash and contingent securities, and can be effectively viewed through the lens of security-bid auctions.[3] Why did Shire make an all-cash offer? Why are most oil tracts neither producing nor under active exploration?[4] More fundamentally, how do contingent bids affect real option exercise? How should a seller design the auctions to trade off rent extraction and incentive provision? What is the role of the seller's commitment to using specific security bids? What securities buyers prefer to offer?

Without a model of auctions of real options, simultaneously addressing these important

[1]ENERCOM Consulting, 360 articles: Central Gulf of Mexico Lease Sale - Come on in, the Water if Fine! (March 21, 2013), http:oilandgas360.comcentral-gulf-of-mexico-lease-sale-come-on-in-the-water-is-fine.

[2]Shire News: Shire to Acquire NPS Pharma as Further Step in Building a Leading Biotech. http://www.shire.com/shireplc/en/investors/irshirenews?id=1052

[3]Oil leases have been auctioned using cash, bonus-bid, royalty and profit-share contracts. In technology transfers, such as the licensing in pharmaceuticals, rivals bid contingent contracts (Vishwasrao (2007) and Bessy and Brousseau (1998)). In sales of large assets, such as the wireless spectrum auction for FCC bandwidth, aggressive bidders can declare bankruptcy and the bids are essentially debts (Board (2007a) and Zheng (2001)). Equities, preferred convertibles, and call options are frequently used in M&A and venture capital financing (Martin (1996), Kaplan and Stromberg (2003), and Hellmann (2006)). Other examples include advance and royalty payments in publishing contracts (Dessauer (1981) and Caves (2003)), motion picture deals (Chisholm (1997)), business licenses such as electronic gambling machines with pre-specified profit tax, and military procurement contracts (McAfee and McMillan (1987b)).

[4]"Oil and Gas Lease Utilization,". Report to the President" by Department of the Interior dated May 2012. 72% of oil and gas tracts offshore and 56% of those that are on federal lands are idle. This revelation has triggered a huge public outcry and heated debate in Congress on the reason for the purported sluggish development of natural resources despite the imbalance in supply and demand, and has policy implications in the backdrop of Obama's proposal to increase onshore royalties by 50%.

questions is difficult. This paper does so by tying together optimal stopping with security bid choices in a tractable framework. Under the intuition that sellers and bidders often derive different values from the option exercise, I show that common security bids distort real option exercises, and characterize the conditions whereby they accelerate or delay investments; I then demonstrate that the seller's revenue is non-monotone in "steepness" — a popular measure for ranking securities in the literature (e.g., DeMarzo, Kremer, and Skrzypacz (2005)) — and security designs can be ranked instead using a mechanism design approach; finally, I find that when a seller lacks commitment to using specific security bids, bidding and option exercise equilibria are equivalent to those in cash auctions.

Auctions of real options are prevalent in licensing and patent acquisitions, leasing of natural resources, real estate development, M&A deals, venture capital and private equity markets, and privatization of large national enterprises.[5] They often entail tremendous financial resources and timing decisions that are prohibitively costly to contract upon sale.[6] After oil and gas leases are sold, it is hard for the government to monitor and contract on the exact time for exploratory and development drills; after an acquisition deal, the target company's shareholders may find it challenging to control when the managers incur irreversible costs to close a plant or launch a new product; real estate developers can typically time the market for construction without the influence of the original landowner who sells the property, even though the latter may retain stakes in the project. I therefore focus on security bidding and the moral hazard of post-auction option exercise, and show their interaction not only adds new theoretical insights, but are relevant in practice as well.

Prior studies on security-bid auctions do not consider how security design influences post-auction investment timing. Similarly, studies on real options analyze the sales and exercise of real options in isolation. This paper therefore attempts to bridge the gap between auction theory and agency theory in real options. Specifically, I model the sale and exercise of a typical investment option with endogenous participation. The baseline model involves a seller

[5] Bolton, Roland, Vickers, and Burda (1992) describe the privatization policies in Central and Eastern Europe. Pakes (1986) and Schwartz (2004) discuss patents as real options.

[6] In the Gulf of Mexico alone, the oil and gas leases auctioned by the U.S. federal government in 1954-2007 have exceeded \$300 billion, and annual licensing deals by pharmaceutical giants exceed \$20 billion even in the aftermath of the financial crisis. M&A volume worldwide is also in the trillions of dollars annually.

and multiple potential bidders who are risk neutral and maximize their expected payoffs. Time is continuous and in two sequential stages. In the first stage, the seller auctions an asset and participants bid cash and contingent securities, before the seller allocates the asset. In the second stage, the winning bidder rationally times the exercise of the investment option and delivers the contingent payment to the seller.

The key friction in the model is the non-contractibility of the bidders' post-auction investment timing, which depends on their private types. Consequently, security bids distort a winning bidder's option exercise from the socially efficient exercises in cash-bid auctions. I first show that for general forms of securities, optimal stopping strategies exist and follow threshold strategies. The acceleration or delay of investments can be simply characterized using the change in what I call "discounted security payoff" — the net present value of a security payoff upon option exercise — evaluated at a given threshold. Unlike earlier studies that focus on the optimal stopping problem associated with debts or equities, this paper characterizes delays and accelerations of option exercises associated with very general forms of securities.

The distortion from the efficient timing can either better or worse align the winning bidder's incentive with the seller's. I show that relative to a social planner, the seller essentially has a real option of which the virtual strike price to her is augmented by an amount equivalent to the information rent she pays, and the exercise is controlled by the winning bidder whose incentives are not aligned with the seller in general. On the one hand, the seller naturally prefers security designs that lead winning bidders to moderate delay the exercise to take this additional exercise cost into consideration. On the other hand, if a security makes the winning bidder either accelerate or excessively delay the option exercise, its cost can dominate the usual benefits of using contingent bids, such as enhanced rent extraction.[7]

I take an initial step in analyzing the tradeoff use a mechanism design approach to decompose the tradeoffs concerning the seller's revenue into considerations about information asymmetry between the seller and the bidder, and those about investment distortions due to

[7] Prima facie, the type of bids should not matter as a cash equivalent always exists. One advantage to contingent bids is that they enhance the seller's revenue by effectively linking payoff to a variable affiliated with bidders' private information—the "linkage" principle in Milgrom (1985). Contingent bids also mitigate liquidity or legal constraints and reduce valuations gaps among various parties.

the use of contingent securities. In particular, the moral hazard breaks the well-established ranking of securities using "steepness" DeMarzo, Kremer, and Skrzypacz (2005). A "steeper" security can link the seller's payoff to the better type and thus allows her to extract greater rent from the bidders, but at the same time it may distort post-auction investment which reduces the total surplus is reduced. In general, the seller prefers security designs that allows her to utilize the information rent (that she has to pay anyway) to motivate the winning bidder to exercise the real option at a time the seller prefers. The optimal design entails a combination of cash and royalty payments, which is consistent with the popular use of negotiated royalty payments and down payments in sales of marketing rights, licensing agreements, publishing and movie contracts, and many other franchise business practices.

I then move on to consider what I call *informal auctions* in which the seller lacks the commitment to security choice and allocation rules. Whereas in formal settings such as oil lease auctions, wireless spectrum auctions, or privatization auctions, the seller specifies explicitly and commits to an ordered set of security bids, sellers in many other sales lack such commitment; that is, bidders bid anything they want and can potentially revise their offers.[8] Prominent examples of informal auctions include corporate takeovers and project finance, where bidders decide what to offer. Still others, such as licensing agreements and contracts in the entertainment industry, appear in both categories.

I show that the seller's commitment to security design significantly influences the bidding and investment outcomes. In particular, bidding equilibria in both first-price and ascending informal auctions are equivalent to those in cash auctions. The intuition is that cash-like bids allow a bidder to generate the maximum social surplus, and at the same time outbid competitors in the cheapest way. For example, a bidder with higher valuation can more easily outbid others using cash versus equity shares, because the same shares cost him more than they cost someone with a lower valuation. This advantage of using cash leads to bidders' using cash in equilibrium, which, despite being suboptimal to the seller, are socially optimal because they remove inefficient post-auction investment.

These results help understand several puzzling empirical observations, and add general

[8]DeMarzo, Kremer, and Skrzypacz (2005) introduce a similar concept but rule out offer adjustments.

theoretical insights to auctions and real option exercises. In particular, a high royalty rate in oil and gas lease auctions causes the winning bidder to delay exploration beyond efficient rational waiting due to optionality, especially in highly uncertain environments, which potentially explains the large number of idle tracts reported. I also show more bidders could decrease revenue and social welfare by exacerbating moral hazard associated with security bids, and how restricting the range of bidding may help mitigate the distortion in investment timing. Finally, I embed my model in a general framework of auctions with post-auction moral hazard to highlight the importance and uniqueness of the timing moral hazard associated with real option exercises, such as the asymmetric distortion of option exercise due to the irreversible nature of time.

Literature

This paper builds on studies of security-bid auctions and their applications in corporate finance.[9] DeMarzo, Kremer, and Skrzypacz (2005) give an extensive exposition of security-bid auctions, showing "steeper" securities lead to higher expected value to the seller. Samuelson (1987) suggests adverse selection and moral hazard complicate the effect. Che and Kim (2010) and Rhodes-Kropf and Viswanathan (2000) demonstrate, respectively, that adverse selection could reverse the ranking of securities and lead to inefficiencies in bankruptcy reorganizations and privatizations. This paper examines post-auction moral hazard—the second issue Samuelson (1987) emphasized. Kogan and Morgan (2010) compare equity and debt auctions under moral hazard in an experimental study. Laffont and Tirole (1987), Eső and Szentes (2007), McAfee and McMillan (1987a), and Riordan and Sappington (1987) also study how moral hazard affects competitive sales. Most closely-related is Wong (2016), which similarly studies post-auction moral hazard, but of information acquisition; so are Povel and Singh (2010) and Liu (2012) that relate security-bid auctions to external financing. This paper is unique in considering the moral hazard of option exercise, incorporating both stan-

[9] Hansen (1985), Crémer (1987) and Riley (1988) are among the early contributions. Hansen (2001) reviews the corporate auction process. Dasgupta and Hansen (2007) and Boone and Mulherin (2007) justify viewing corporate takeovers as auctions. Eckbo, Giammarino, and Heinkel (1990) consider the role of non-cash bids. Skrzypacz (2013) gives an overview.

dard security payments and issues concerning commitment to security design, which are commonly observed in real life, for example, in corporate mergers and acquisitions.

Also related are the theories of incentive contracting, typically applied to defense procurement (Engelbrecht-Wiggans (1987), McAfee and McMillan (1986), and Laffont and Tirole (1987)). Board (2007b) derives optimal selling mechanisms of options. This paper extends by deriving the optimal design under interdependent values in continuous-time setting with persistent private information, and differs primarily in the focus on standard security bids and the associated post-auction real option exercises in both formal and informal auctions, rather than the optimal design.

This study also complements the emerging literature on agency issues in a real-options framework. Maeland (2002), Grenadier and Wang (2005), and Cong (2012) study distortion of investment incentives due to adverse selection and moral hazard. Grenadier and Malenko (2011) study how firm managers accelerate or delay real option exercise to signal private information. Other studies on dynamic agency and investment under uncertainty involve either first-best effort being optimal even with moral hazard (e.g. DeMarzo and Fishman (2007)), or the irrelevance of real option exercise under the first-best scenario (e.g. Philippon and Sannikov (2007)). One exception is Gryglewicz and Hartman-Glaser (2015) that shows the magnitude of effort cost and severity of moral hazard affect real option timing in a non-monotone fashion. This paper complements and illustrates that accelerations and delays in real investments can also arise due to security bidding and seller's endogenous auction design. The asymmetric distortion due to the irreversible nature of time complements studies such as Grenadier and Malenko (2011) and Cong (2017) to underscore the uniqueness of corporate timing decisions.

The remainder of the paper proceeds as follows. Section 2 describes the economic environment and sets up the model; Section 3 analyzes optimal strategies for post-auction option exercise. Section 4 derives bidding equilibria and security ranking in formal auctions. Section 5 characterizes informal auctions as games of signaling and investment timing. Section 6 discusses further implications and extensions. Section 7 concludes. The appendix contains all the proofs and generalizations.

2 Setup and Optimal Exercise

A risk-neutral revenue-maximizing seller with discount rate $r > 0$ owns a project with an embedded option. Once developed, the project generates a verifiable lump sum cash flow whose value P_t is public and evolves stochastically according to a geometric Brownian motion (GBM)

$$dP_t = \mu P_t dt + \sigma P_t dB_t, \tag{1}$$

where B_t is a standard Brownian motion under the equivalent martingale measure, μ is the instantaneous conditional expected percentage change per unit time in P_t, and σ is the instantaneous conditional standard deviation per unit time. I assume $\mu < r$ to ensure a finite value of the option. Note that we could interpret P_t as the present value of a stream of future cash flows.

The seller does not have the expertise to exploit the option but can auction the project to N risk neutral potential bidders with the same discount rate r who have the expertise to exploit the option.[10] Bidder i knows his private investment cost for the project θ_i, and the distribution of types for other bidders which is i.i.d. with positive support $[\underline{\theta}, \overline{\theta}]$.[11] Denote the cumulative distribution and density function by $F(\theta)$ and $f(\theta)$, respectively. Similar to DeMarzo, Kremer, and Skrzypacz (2005) (DKS), a bidder has to pay a cost $X \geq 0$ upon winning the auction, which we can interpret as the initial resources the project requires, such as illiquid human capital, the social cost of underwriting securities, or simply his opportunity cost.[12] Readers should interpret X as expenses that have to be incurred upfront, which DKS does not require in their static setup. It mainly endogenizes the bidder participation, and is not crucial for our results. The project is worthless to the bidder if it is never developed.

I assume that whereas the revenue from exercise P is observable and contractible, the cost θ and thus the profit $P - \theta$ are not. This assumption is realistic because contracts or security

[10]For clarity, I refer to the seller as female and the bidders as male.

[11]The analysis also applies when bidders differ in other quantities, such as production capacities.

[12]DKS remarks in the extension that the moral hazard associated with investment X can help rule out reimbursement from the seller (which violates limited liability) in equilibrium. I focus on the moral hazard of post-auction exercise of the real option even with regular securities that satisfy limited liability, and how it distorts bidding and investment efficiency.

designs based on profit are rare due to high monitoring costs, limited comparability, and landowners' risk aversion (Robinson (1984)). The procurement literature has also established that profit reporting is subject to manipulations, and contracting on revenue is more feasible. Past experience in oil lease auctions has also shown considerable difficulties in reaching agreement on the proper profits (Opaluch, Grigalunas, Anderson, Trandafir, and Jin (2010)). Consequently, payments are usually contingent on top-line revenue in the development of natural resources, contracts on marketing and licensing rights, as well as franchise chain operations.

Bidders compete by offering security bids that are combinations of contingent payments from the cash flow of the project and non-contingent payments that can be viewed as upfront cash. Unless stated otherwise, the remainder of the paper focuses on standard security bids as defined next.

DEFINITION. *A **standard security bid*** is an upfront cash payment $C \in \mathbb{R}$ and a contingent payment at the time of investment τ given by an upper-semicontinuous, weakly increasing function $S(P_\tau) \in [0, P_\tau]$.

Upper-semicontinuity is a very weak technical condition to ensure the existence of optimal stopping solution, and is satisfied by almost all commonly used securities. $S(P_\tau) \in [0, P_\tau]$ implies limited liability from both the seller and bidders. Standard security bids are simple and intuitive, and as discussed later, can implement the optimal auction design even in the augmented universe of security bids. They admit, for example, equity bids for which the seller receives a fraction α of the payoff ($S(P) = \alpha P$), call option bids for which the seller can pay a strike price k for the project cash flow ($S(P) = [P - k]^+$), and bonus bids on fixed royalty rate ϕ for which the seller receives bonus C and royalty payment $S(P) = \phi P$.

The agents interact in continuous time in two stages: the auction stage, and the post-auction timing stage. I work backward to first solve for the optimal investment strategy for the winning bidder, then derive the bidding equilibrium given the bidders' valuations based on their investment strategies.

Formal auctions and informal auctions mainly differ in the seller's commitment to the form of security bids. Throughout the paper, I focus on first-price auctions (FPAs) and second-price auctions (SPAs) in which the bidder with the highest bid wins and pays the highest bid or the second-highest bid, respectively. I assume the seller commits to no renegotiation post-auction, and to no contracting or resale to losing or non-participating bidders.

Welfare is defined as the total payoff to the seller and bidders, and efficiency in this paper means constrained efficiency from a global optimizer's perspective; that is, welfare maximizing under the same informational or institutional constraints as individual agents.

Cash Auctions as a Benchmark

In cash auctions, a bidder of type θ owns the project entirely upon winning, and optimally develops the project at time t to maximize $\mathbb{E}[e^{-rt}(P_t - \theta)]$. The optimal strategy for this standard problem is well-known and involves immediate investment upon reaching an upper threshold $P^*(\theta)$ (e.g., McDonald and Siegel (1986) and Dixit and Pindyck (1994)). The value of the investment option W and $P^*(\theta)$ are independent of X and t, and are given by

$$P^*(\theta) = \max\left\{P_0, \frac{\beta}{\beta-1}\theta\right\}, \quad \textit{where} \quad \beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \quad \textit{and} \tag{2}$$

$$W(P_0;\theta) = D(P_0; P^*(\theta))(P^*(\theta) - \theta), \quad \textit{where} \quad D(P;P') = \left(\frac{P}{P'}\right)^{\beta} \quad \textit{for} \quad P \leq P'. \tag{3}$$

Note that $D(P_t; P')$ corresponds to the time-t price of an Arrow-Debreu security that pays one dollar when the first moment threshold $P' \geq P_t$ is reached, also known as the "expected discount factor". The option value of the project is simply the total value of Arrow-Debreu securities that replicate the payoff of the investment option at exercise.

Bidder i's private valuation is then $W(P_0;\theta_i) - X$, which decreases in θ_i, and his bidding strategies are the same as those in standard cash FPAs and SPAs. A cutoff type exists for participation $\theta_c = \min\{\overline{\theta}, \theta_{BE}\}$, where the break-even type θ_{BE} solves $W(P_0;\theta) - X = 0$ and is given explicitly as $\theta_{BE} = (\beta-1)(P_0^{\beta}\beta^{-\beta}X^{-1})^{\frac{1}{\beta-1}}\mathbb{1}_{\{P_0>\beta X\}} + (P_0 - X)\mathbb{1}_{\{P_0\leq\beta X\}}$. Types with costs higher than θ_c do not participate. Because post-auction investments are not distorted, FPAs and SPAs generate equivalent revenues to the seller (Revenue Equivalence Theorem), and efficiently allocate the project to type $\theta_{(1)}$ if $\theta_{(1)} \leq \theta_c$, where $\theta_{(j)}$ is the jth lowest realized θ. The cases with a reserve price or entry fee are similar.

3 Optimal Post-sale Investments

Whereas earlier studies involving real option exercises typically focus on simple forms of security bids, such as equities or cash, which admits well-known optimal stopping solutions, our security space is significantly more general. As such, a priori, there is no guarantee that optimal value function and investment strategy exist and are well-behaved. Adding to the difficulty is that most theorems and results in the studies of optimal stopping that the extant real options literature reference involve a payoff function being $\mathcal{C}^1$ (see, for example, Strulovici and Szydlowski (2012) for a recent discussion). However, securities commonly used in financial markets often have kinks, which makes deriving optimal stopping strategies in our setting rather challenging.

Therefore, I first characterize the optimal value function and the corresponding investment strategy, taking as given the standard security bid in equilibrium. Allowing security bids in general forms complicates the solution to the optimal stopping problem, because it can no longer be taken for granted that the solution is a simple first-hitting problem.

Suppose the winning bidder of type θ pays standard security $\{C, S(P_t)\}$ where C is paid at the time of the auction and $S(P_t)$ at time t of investment option exercise. His private valuation at the auction is

$$\tilde{V}(C, S(\cdot), \theta) = \sup_{\tau \geq 0} \mathbb{E}_P[e^{-r\tau}(P_\tau - S(P_\tau) - \theta)] - X - C, \tag{4}$$

where τ is the stopping time that he optimizes over. The fact that $S(P)$ is of general form distinguishes this problem from traditional real-options models. The following lemma dispels the concern that $\tilde{V}(\theta)$ may not be well-defined.

Lemma 1. *An optimal value function $\tilde{V}(C, S(\cdot), \theta)$ as defined in 4exists and is continuously* *decreasing in θ. The optimal stopping time involves P first hitting a threshold value from either above or below.*

Notice that even with potential jumps in the security payoff, the optimal value function is still well-defined. Consistent with Theorem 5.10 in Harrison (2013), an optimal stopping strategy generally involves first-hitting and may have both upper and lower thresholds that are dependent on P_0.

Following the convention in the literature of security design, I further assume,

Assumption 1. *In formal auctions, standard securities also satisfy double-monotonicity: both $S(P)$ and $P - S(P)$ are weakly increasing in P.*

In other words, in addition to being standard, a security bid also satisfies that $P - S(P)$ is weakly increasing. Double-monotonicity is a standard assumption in security design (e.g., Harris and Raviv (1989); Innes (1990); Nachman and Noe (1994); Hart and Moore (1995a); DeMarzo and Duffie (1999); DeMarzo, Kremer, and Skrzypacz (2005)), and is satisfied by most securities and contracts used in practice, such as equity, warrants, convertible debt, and even uncommon securities such as putable stock.[13] Moreover, this assumption allows us to focus on one-sided threshold strategies that the literature typically studies. I relax this assumption when discussing informal auctions in which potential bidders can offer any standard security with the property that $S(P)$ is weakly increasing.

Proposition 1. *Under Assumption 1, $P^{-\beta}[P - S(P) - \theta]$ has an optimizer $\tilde{P} \geq P_0$ for all participating bidders. An optimal investment strategy exists and follows a threshold strategy involving $\tau(\tilde{P}) = \{\inf t \geq 0 : P_t \geq \tilde{P}\}$. Consequently, the optimal value function is given by* $\sup_{P \geq P_0} D(P_0; P)[P - S(P) - \theta] - X$.

The optimal stopping obviously depends on P_0 and the security used. In fact, we show in the proof that $\tilde{V}(C, S(\cdot), \theta)$ is non-decreasing in P_0.[14] More importantly, the proposition provides us with a simple way to characterize optimal investment for any given security in

[13] As is argued in the literature, if "charitable contributions" and "burning cash" cannot be prevented, then the assumption is without loss of generality.

[14] This implies that auction timing matters. I model auction timing in an earlier draft of the paper, and give more detailed discussion in a companion paper Cong (2017).

closed-forms without imposing ad hoc restrictions, as illustrated next for the case of equity bids.

Equity Bids and Investment Delays

For equity bids, $C = 0$ and $S(P) = \alpha P$ given a bid of α which is the fraction of shares the winning bidder has to pay. We show later that bidding equilibria exist for both FPAs and SPAs. Proposition 1 implies that the optimal investment threshold can be derived from the first- and second-order conditions,[15]

Corollary 1. *In auctions with equity bids, the winning bidder invests when cash flow first reaches* $P^{equity}(\theta) = \max\left\{P_0, \frac{\beta\theta}{(\beta-1)(1-\alpha)}\right\}$.

Comparing the threshold to $P^*(\theta)$, the investment is inefficiently delayed—undesirable to the seller because her revenue $D(P_0; P)S(P)$ is decreasing in P. Ex post the auction, investing some time earlier could improve both seller's revenue and welfare.

Existing real-option models with agency, such as that in Grenadier and Wang (2005), also predict decreased or delayed investments, as we see here. However, the security choice in selling real options could also lead to accelerated investment, and can be an alternative to empire-building-based explanations of overinvestment, as the next example illustrates.[16]

Call Option Auctions and Investment Accelerations

Consider call option bids with no reserve price. Let k be the strike price the winning bidder of type θ contracts, then $S(P) = \max\{P - k, 0\}$ and $C = 0$. Bidder θ's present value conditional on winning and exercising at τ is $\mathbb{E}_P[D(P_0; P_\tau)(P_\tau - \max\{P_\tau - k, 0\} - \theta)] - X$. It is a dominated strategy for type θ to bid a strike less than $X + \theta$, because if he fails to break even upon winning. So he is better off bidding $k \geq X + \theta$. If he bids a strike greater than $P^*(\theta)$, upon winning in either FPA or SPA he always invests with the threshold $P^*(\theta)$, and the seller's call option is never exercised. But then he could bid a lower k to increase

[15]In the appendix I provide a standard verification argument, which works for equity bids, to highlight how Proposition 1 simplifies the characterization, not to mention that the standard verification argument does not work for many other securities without further technical assumptions.

[16]Gryglewicz and Hartman-Glaser (2015) underscore this point in a setting with dynamic agency.

the chance of winning. Hence $k \in [X+\theta, P^*(\theta)]$. The investment threshold maximizes the winning bidder's value, as described in the next corollary.

Corollary 2. *In auctions with call option bids, a bidder of type θ always bids $k \in [X+\theta, P^*(\theta)]$, and upon winning, invests when the cash flow first reaches $P^{call}(\theta) = \max\{P_0, k\}$.*

Notice $P^{call}(\theta) \leq P^*(\theta)$ and the equality holds when $P_0 > \frac{\beta}{\beta-1}\theta$. Inefficiency therefore lies in the potential *acceleration* of investments. Basically, if the call option is going to be exercised, there is no incentive for the bidder to keep timing the market because doing so delays his payment k.

Figure 1 illustrates how different security bids lead to different investment thresholds and timings. The figure includes another common form of security: friendly debt $S(P) = \min(P, B)$, where B is a fixed promise of payment.[17]

Discounted Securities and Optimal Investment

It turns out that we can go beyond equity and call option to say more about optimal investment under standard securities in general. Compared to the investment threshold $\frac{\beta}{\beta-1}\theta$ in Equation (2), the threshold with security payment can be higher because the bidder faces an additional cost $S(P)$. However, the sensitivity of the security payment to cash flow implies a smaller option premium. Depending on which effect dominates, the threshold could be either higher or lower. We next provide a simple general characterization of investment distortion.

Proposition 2. *Suppose a winning bidder optimally invests at P^*. Then relative to the socially efficient post-auction investment timing, the investment is strictly delayed if $P^{-\beta}S(P)$ is increasing at P^*, and weakly accelerated if $P^{-\beta}S(P)$ is weakly decreasing at P^*.*

We note that investment timing is socially efficient in a cash-bid auction, thus the proposition is essentially a characterization of how security bids distort post-sale optimal exercise of the

[17]Friendly debts are essentially zero-coupon debts, also known as Qard/Qardul hassan in Islamic finance. They are popular in contractual agreements in Islamic banking and microfinance, and are equivalent to granting the winning bidder instead of the seller call options - the exact opposite situation to that for call option bids.

real option. This proposition does not require knowing the type of the winning bidder and shows the distortion only depends on the change in "discounted security" $P_0^\beta P^{-\beta} S(P) = D(P_0; P)S(P)$ evaluated at the equilibrium investment timing. Given that security bid in equilibrium is publicly observed, the proposition is a simple and practical characterization of the investment distortion. In the appendix, I also prove two complementary results, summarized in the next corollaries.

Corollary 3. *A winning bidder of type θ strictly delays investment if $P^{-\beta}S(P)$ is decreasing at $P^*(\theta) = \frac{\beta}{\beta-1}\theta$, and weakly accelerates investment if $P^{-\beta}S(P)$ is weakly increasing at $P^*(\theta)$.*

Corollary 4. *For any security such that the function $P^{-\beta}S(P)$ is unimodal in $[P_0, \infty)$, the investment is always inefficiently and weakly accelerated if $P^{-\beta}S(P)$ has a mode smaller than $P^*(\underline{\theta})$; the investment is always inefficiently and weakly delayed if $P^{-\beta}S(P)$ has a mode greater than $P^*(\overline{\theta})$, and the delay is strict if the mode(s) of $P^{-\beta}S(P)$ are all greater than $P^*(\overline{\theta})$.*

Note that unimodality of the discounted security function is satisfied by many commonly used securities.

When we change the exercise threshold, it is the corresponding change in discounted security payoff that matters. If as we increase the exercise threshold, the option value increases less than the discounted security payment to the bidder, the latter is (locally) biased towards delaying the exercise relative to a social planner; otherwise, he is biased towards accelerating the investment.

What is more interesting is the asymmetry in delay versus acceleration: investment is weakly accelerated in Corollary 4 even if the mode(s) of $P^{-\beta}S(P)$ are all smaller than $P^*(\underline{\theta})$. This asymmetry also appears in Proposition 2 in that even if $P^{-\beta}S(P)$ is strictly decreasing at optimal exercise or strictly increasing at $P^*(\theta)$, we cannot conclude the investment is accelerated. This asymmetry derives from the fundamental irreversibility of time: for an auction held at threshold P_0, one cannot deviate to exercising the option at a first-hitting threshold smaller than P_0 (going back in time), but can always deviate to exercising at a first-hitting threshold higher than P_0 (waiting longer). In other words, if the socially efficient investment timing is at the time of the auction, a winner who prefers to accelerate the investment can at best invest at P_0 but not earlier.

4 Formal Auctions and Ranking Securities

Given the post-auction exercise of the real option, this section describes and analyzes formal auctions. I first derive the bidding equilibria, then illustrate how conventional results on security ranking are modified, before providing conditions for comparing different types of securities. Later in Section 6 I relate the findings to the auctions and development of oil and gas tracts, and discuss optimal auction design.

In ***formal auctions***, the seller commits to a pre-specified, well-ordered set of allowed bids, which are ranked by simple, easily implementable rules. A variant of the definition in DKS formalizes this notion of well-orderedness:

DEFINITION An ***ordered set of securities*** ranked by index s is defined by a left-continuous map $\Pi(s) = \{C(s), S(s,\cdot)\}$ from $[s_L, s_H] \subset \mathbb{R}$ to the set of standard security bids such that for each voluntary participant of type θ, $\boldsymbol{V(s,\theta) \equiv \tilde{V}(C(s), S(s,\cdot), \theta)}$ is non-negative and non-increasing in s on $[s_L, \tilde{s}]$ and negative on $(\tilde{s}, s_H]$ for some $\tilde{s} \in [s_L, s_H)$.

This definition simply requires that in addition to being standard, an ordered set of securities admits one-dimensional ranking with index s for any payoff from the project, and permissible bids cover a wide range such that each participant earns a non-negative profit by bidding low enough but earns no profit by bidding too high. Any such sets can be represented by the mapping defined above up to an order-preserving transformation of the index. $C(s)$ really captures the payment that is not contingent on the bidder's investment timing action, and $S(s,\cdot)$ is the contingent component (where we have a moral hazard due to contract incompleteness). In fact, $C(s)$ does not play a significant role in the distortion of investment timing, and is mainly used in the discussion of optimal mechanism design to satisfy the incentive compatibility constraints. One can simply think of a typical ordered set of securities as having one component fixed when the other component is allowed to vary, which are common in practice, or the two component covary positively so that the ranking is unambiguous. The current specification nests these cases, and also allows other cases as long as the securities with the set can be ordered.

This notion of an ordered set of securities subsumes a definition based on securities' values to the seller for a given type of the bidder, because the latter is necessarily an ordered set here. Our definition is more inclusive of contingent bids used in real life: s could be the fraction of shares α in a pure equity auction $\{C(\alpha) = 0, S(\alpha, P) = \alpha P\}$, the (negative) strike price k in a call-option auction $\{C(-k) = 0, S(-k, P) = \max\{P - k, 0\}\}$, or the bonus b in a bonus-bid auction with royalty rate ϕ fixed $\{C(b) = b, S(b, P) = \phi P\}$.[18] M&As, VC contracts, and lease auctions routinely use such securities, and indeed the bidder offering the highest s wins.[19]

To focus on monotone separating equilibria, we assume the seller commits to allocating the project to the bidder with the highest index. In fact, most security bids I examine including the optimal security derived later are also well-ordered in terms of their values to the seller. Had we restricted our attention to an ordered set of securities where high s gives higher value to the seller, we still need to impose the above condition in the definition for a bidding equilibrium to exist. The winning bidder pays a security using the highest-bid index in FPAs or the next-highest-bid index in SPAs. This is common in practice and in earlier models (e.g., Kogan and Morgan (2010)). If it were not the case, the better types cannot always separate and we may have partial pooling, which I discuss in Section **??**.

4.1 Equilibrium Bidding and Allocation

Using the fact that $V(s, \theta)$ is well-defined (Lemma 1), I characterize the auction equilibria (counterparts to DKS Lemmas 2 and 3 but for auctions of real options). Throughout the

[18] One could equivalently define an ordered set based on monotonicity of security values to the seller and derive almost all the results, but that approach rules out auctions with commonly used security bids, such as equity bids without a minimum share retention.

[19] In M&As with the acquirer's stocks as bids, C simply corresponds to the value of the acquirer's cash flows that are independent of the acquisition, X corresponds to the opportunity cost of incorporating the target firm, and P is the payoff from the acquired assets and projects, and the synergy created.

paper, I assume that the standard "single-crossing" (as described in Lemma 2) for FPAs. I also assume that buyers resolves any indifference in bidding by bidding the higher s.

Lemma 2. *In FPAs, when $\ln V(s,\theta)$ is absolutely continuous in s with the derivative (when exists) decreasing in θ, a unique symmetric Bayesian Nash equilibrium exists that is decreasing, differentiable, and is characterized by:*

$$s'(\theta) = \frac{(N-1)f(\theta)}{1-F(\theta)} \frac{V(s(\theta),\theta)}{V_1(s(\theta),\theta)} \tag{5}$$

for $\theta \leq \hat{\theta}$ with the boundary condition $s(\hat{\theta}) = \sup\{s \in [s_L, s_H] \mid V(s,\hat{\theta}) = 0\}$. The cut-off type for participation is $\hat{\theta} = \sup\{\theta \leq \overline{\theta} \mid \max_s V(s,\theta) \geq 0\}$.

Lemma 3. *In SPAs, the unique Bayesian Nash equilibrium in weakly undominated strategies is for type θ to bid $s(\theta) = \sup\{s \in [s_L, s_H] \mid V(s,\theta) \geq 0\}$, which is decreasing in θ. The cut-off type for participation is $\hat{\theta} = \sup\{\theta \leq \overline{\theta} \mid \max_s V(s,\theta) \geq 0\}$.*

The fact that the bidding strategies are monotone implies directly that the investment option should be allocated, if at all, to a bidder with the least investment cost. The level of participation is the same for FPAs and SPAs, and is weakly smaller than that in cash auctions. In addition, the amount of competition as indicated by N, the initial commitment cost X, and the cashflow level at the time of the auction P_0 all affect bidding behavior:

Corollary 5. *Bidders bid more aggressively (weakly greater s for all types, and strictly greater s for a positive measure of types) in FPAs with security bids as N increases or X decreases, or if V and V/V_1 are increasing in P_0, as P_0 increases. They bid more aggressively in SPAs with security bids as X decreases or if V is increasing in P_0, as P_0 increases .*

Intuitively, a smaller X or higher P_0 correspond to higher valuations of the project by the bidders, which allows them to promise more to the seller to increase their chances of winning. When N is bigger in FPAs, one has to increase the bid to outbid more competitors. However, this does not apply in SPAs because on bidders' bidding strategy is independent of others' bids.

Take equity auctions for example. In SPAs, the bidder θ increases α until $V(\alpha, \theta) = 0$. In FPAs, since $\frac{\partial^2 \ln V(\alpha,\theta)}{\partial\alpha\partial\theta} < 0$ is well-defined except on the boundary $P_0 = \frac{\beta\theta}{(\beta-1)(1-\alpha)}$, Lemma 2 applies and $\alpha(\theta)$ is continuous and decreasing. For example, when $X = 0$,

$$\alpha(\theta) = \int_{\theta}^{\hat{\theta}} \frac{(N-1)f(\theta'')}{\beta(1-F(\theta''))} \exp\left[\int_{\theta''}^{\theta} \frac{(N-1)f(\theta')}{\beta(1-F(\theta'))} d\theta'\right] d\theta'', \quad \textit{for} \quad \theta \leq \hat{\theta}. \tag{6}$$

With uniform distribution, this translates to $\alpha(\theta) = 1 - (\frac{\overline{\theta}-\hat{\theta}}{\overline{\theta}-\theta})^{\frac{N-1}{\beta}}$. Clearly bidders bid more shares when N or P_0 increases or X decreases (cutoff $\hat{\theta}$ is increasing in P_0 and decreasing in X). Note inefficient delays of investments also follow directly from Corollary 1.

The bidding with call options is similar. In FPAs, when $P_0 \leq k$, $V = \frac{P_0^\beta (k-\theta)}{k^\beta} - X$, otherwise $V = k - \theta - X$. Lemma 2 applies for the bidding equilibrium.[20] Next for SPAs, for those who participate, they bid up to their valuations, in other words, $k = \theta + X$ if $\theta < P_0 - X$ or k solves $\frac{P_0^\beta (k-\theta)}{k^\beta} = X$ otherwise. In either case, $k < \frac{\beta}{\beta-1}\theta$ and the investment is strictly accelerated. In fact, the seller makes a profit only when $\theta < P_0 - X$, otherwise the strike price is simply the value of the project, netting her no profit. The cutoff type is the same as that in FPAs.

With equilibrium bidding and allocation characterized, we next examine how some of the conventional results on security ranking are modified.

4.2 Security Ranking

While the bidding equilibria resemble those in DKS, bidders' incentives for option exercise generally differ from the seller's, and this misalignment alters many conventional results in security-bid auctions. Below I first give some examples for which the results in DKS no longer apply. I then discuss why the tradeoff considered here is different from that in DKS and is new to the literature, before taking a mechanism design approach to ranking the securities.

[20] Note $\frac{\partial \ln V}{\partial(-k)} < 0$. For $k \leq P_0$, $\frac{\partial^2 \ln V}{\partial(-k)\partial\theta} = -\frac{1}{(k-\theta-X)^2} < 0$; for $k > P_0$, it is $-\frac{(\beta-1)}{V^2 k^\beta}\left[\left(\frac{P_0}{k}\right)^\beta \frac{\theta}{\beta-1} - X\right] \leq -\frac{(\beta-1)}{V^2 k^\beta}\left[\left(\frac{P_0}{k}\right)^\beta (k-\theta) - X\right] = -\frac{(\beta-1)}{V k^\beta} < 0$ using the fact $k \leq \frac{\beta}{\beta-1}\theta$.

No “One-Size-Fits-All”

DKS show that “steeper” securities yield higher revenues for the seller. This ranking can break down due to post-auction moral hazard: a “steeper” security extracts more from the winning bidder’s information rent, but it may also reduce his incentive to invest efficiently post-auction. This is also seen in figures 4 and 3 and the table below. In general, there is no “one size fits all” for security ranking. As I show in this section, not only the “steepness” of a security has to be traded off against the incentive provision for option exercise, but the exercise timing distortion alone causes interesting tradeoffs in the seller’s revenue as well. Ranking depends on parameters such as N and σ, such as shown in Figure 3. Ranking also depends on the initial cash flow P_0. as seen in Figure 4(a): among several pure contingent securities, equity gives the highest expected revenue and call option the lowest at $P_0 = 280$, whereas call option is the highest and debt is the lowest when $P_0 = 360$. The worst security design when $P_0 = 300$ more than doubles the revenue from the best security design when $P_0 = 220$. Welfare is similarly affected (Figure 4(b)). In this regard, strategic timing is as important as security design, a topic explored in Cong (2017). In sum, security ranking has to be considered in conjunction with potential misalignment of incentives, timing of auctions, and the number of bidders.

To illustrate how the investment timing make security bids either benefitial or costly, consider the following example of security-bid SPAs. We include bonus-bid auctions to relate to oil lease auctions. Though a bonus-bid auction with $\phi = 1/8$ results in higher revenue, but welfare is significantly lowered due to inefficient investment. Moreover, both equity bids and bonus bids with $\phi = 1/4$ yield significantly lower revenues and welfares compared to the cash auction.

$P_0 = 15$, $N = 18$, $\beta = 1.8$, $\theta \sim Unif[10, 50]$, $X = 3$.

Security Design	Cash-bid	Equity-bid	Bonus-bid $\phi = 1/8$	Bonus-bid $\phi = 1/4$
Revenue	1.6578	1.6416	1.7403	1.4429
Welfare	2.2565	2.0597	2.1862	1.6027

Table 1: Expected Welfare and Seller’s Revenue.

"Steepness of Discounted Securities"

There are at least two approaches to the challenging problem of security ranking under moral hazard. While not particularly straight-forward, one way to rank the security types is to use a modified concept of "steepness" in DKS. The benefit of security bids in DKS relies on the linkage principle which operates through the fact that for a given type of the bidder, the security payoff depends on the value of the asset (type) not only through the action the winning bidder takes. In our setting the security payoff only depends on the timing action. Therefore, the tradeoff in using security bids in our setting is fundamentally different from that in DKS, and the linkage principle is not the driving force in our setting and the "steepness" approach does not apply directly.

That said, we can still derive sufficient conditions for a security design to be "steeper" than another. If the discounted security payoffs after considering the moral hazard of investment timing can be ranked in terms of "steepness", the tradeoff highlighted in DKS would dominate.

Let S and $\tilde{S}$ denote two sets of ordered standard securities (basically they denote the types of securities) indexed by s and $\tilde{s}$ respectively. Let the optimal investment thresholds under the two sets be $P(\theta, s)$ and $\tilde{P}(\theta, \tilde{s})$, where in the notation it is explicit that the threshold depends on the exact security used (both the type and the index), and the type of the bidder who makes the investment decision. Following DKS, I define a discounted security $D(P_0; P(\theta, s))S(P(\theta, s))$ to be "steeper" than $D(P_0; \tilde{P}(\theta, \tilde{s}))\tilde{S}(\tilde{P}(\theta, \tilde{s}))$ if

(1) S and $\tilde{S}$ are continuous in P.

(2) $\forall\theta$, s, and $\tilde{s}$, $P(\theta, s)$ and $\tilde{P}(\theta, \tilde{s})$ are continuous in θ.

(3) $\forall\theta$, whenever $C(s) + D(P_0; P(\theta, s))S(P(\theta, s)) = \tilde{C}(\tilde{s}) + D(P_0; \tilde{P}(\theta, \tilde{s}))\tilde{S}(\tilde{P}(\theta, \tilde{s}))$, $D(P_0; P(\theta, s))S(P(\theta, s)) - D(P_0; \tilde{P}(\theta, \tilde{s}))\tilde{S}(\tilde{P}(\theta, \tilde{s}))$ is increasing at θ.

The conditions basically require that securities from the set $\{C, S\}$ be more sensitive to the true type at the crossing point whenever the NPV of a security from $\{C, S\}$ and one from $\{\tilde{C}, \tilde{S}\}$ are equal. This is similar to the "steepness" requirement in DKS, but applied to the properly discounted value of the security payoffs, where the discounting is endogenously determined by the optimal option exercise. If the discounted security of S is steeper than that of $\tilde{S}$, $\{C, S\}$ generates weakly more revenue to the seller than $\{\tilde{C}, \tilde{S}\}$. Unfortunately, while steepness in DKS allows ranking of most common securities, the modified version of "steepness" only does so when we impose further parametric conditions on some common securities. Therefore, I take a mechanism design approach to write out the seller's expected revenue directly.

A Mechanism Design Approach

The direct revelation principle allows us to focus on a truth-telling mechanism. Suppose the seller times the auction at t_a and specifies the security choice of the general form $S(\tilde{\theta}_i, \theta_{-i}, \mathcal{I}_t)$ at time $t \geq t_a$, where $\tilde{\theta}_i$ is the reported type by i, θ_{-i} are other participants' reported types, and $\mathcal{I}_\tau$ is the set of contractible information up to time t.[21] I assume for the remainder of the paper that $z(\theta) = \theta + F(\theta)/f(\theta)$ is increasing.[22]

I derive in the appendix an integral form of the bidders' incentive compatibility conditions, and in so doing extend Board (2007b)'s analysis to standard real-options settings with standard security bids and even interdependent values.

Proposition 3. *The seller's revenue in formal FPA and SPA with standard security bids is given by*

$$\mathbb{E}\left[\mathbb{1}_{\{\theta_{(1)} \leq \hat{\theta}\}}\left[e^{-r\tau_1^*}(P_{\tau_1^*} - z(\theta_{(1)})) - X\right]\right], \tag{7}$$

where $\theta_{(1)}$ is the smallest realized cost, and τ_1^* is the bidder's corresponding optimal stopping time for investment according to Proposition 1, and Lemmas 2 and 3, with the cutoff type $\hat{\theta}$ given therein.

The seller's payoff thus depends on the "virtual valuation" of the best type rather than the actual valuation.[23] The seller essentially owns the best type's real option with an addi-

[21] Standard security bids is a special case under this general form. Though written in flow payment, S could be a lump-sum payment when it is a Delta function. For standard security bids, $\mathcal{I}_\tau$ contains cash flow from project P_τ when invested at τ, but in general $\mathcal{I}_t$ could include the history of P up to t, and t itself if they are contractible.

[22] This assumption is standard in the auctions literature, for example, see Krishna (2009). One sufficient condition is the "inverse hazard function" $F(\theta)/f(\theta)$'s being non-decreasing. The general intuition still applies without this assumption, though one has to introduce "ironing" techniques which complicates the discussion.

[23] This payoff is equivalent to the expected marginal revenue (MR), see Bulow and Roberts (1989).

tional stochastic cost $F(\theta_{(1)})/f(\theta_{(1)})$. Consequently, the winning bidder's optimal investment timing differs from what the seller would prefer.

The mechanism design approach allows us to write out the seller's expected payoff, which interacts with post-auction option exercise only through τ^*. Therefore we can separately analyze the information cost the seller has to pay, which is common in auction bidding equilibria when bidders are privately informed, from how post-auction exercise impacts the seller's revenue. The information rent affects how the seller and bidders split the surplus, and the investment distortion due to security design affects the total surplus generated. In general, the seller prefers a later option exercise than what is socially optimal because $P^*(z(\theta)) > P^*(\theta)$.

This proposition applies to ranking all ordered standard securities, and in a sense is more general than the results in DKS which only applies to sets that can be compared in terms of steepness. Given a set of securities and the prior on bidder types, Proposition 1 allows us to compute the investment distortion for each type, which, together with Proposition 3, allows us to compute the expected revenue the seller gets. We therefore can compare two security designs easily in FPAs and SPAs. In particular, a contingent bid that is proportional to the reverse hazard rate would generate a higher revenue for the seller, which I further discuss when solving for the optimal design in Section 6.4.

So far, we have assumed that the seller can commit to the security design and allocation rule. This assumption does not hold in many real life situations where the seller always choose the most attractive offer ex post, potentially allowing bids outside the pre-specified set of securities. I turn to this case next.

5 Informal Auctions and the Cash Advantage

Many economic interactions such as corporate takeovers, competition for supply contracts, and talent recruitment have characteristics of auctions because buyers are competing with one another to make offers.[24]. Yet unlike formal auctions in which the sellers restrict

[24] When an asset of a Delaware corporation is for sale, the Revlon rule imposes upon directors a duty to solicit competitive bids to maximize shareholders' value. It may seem that many takeovers occur after

security bids to a pre-specified ordered set, bidders often come up with their own offer terms and the sellers typically lack commitment to explicit auction design, taking the most desirable offer ex post. For example, in Shire's acquisition of NPS Pharmaceutical, Shire repeatedly proposed various deal terms and NPS was considering deals with other pharmaceutical companies as well. I follow DKS to call these transactions informal auctions.[25] What security design emerges in equilibrium? Is there still inefficient option exercise in informal auctions? Answers to these questions would allow us to apply the insights derived earlier to a broader array of auctions of real options in real life.

I find investments are always efficient when the seller cannot commit to pre-specified security design, and in equilibrium, every bid is equivalent to cash. This strengthens the conclusion in DKS: cash is not only the cheapest way for a better type to separate from worse types; it is also the most efficient way.

5.1 A Signaling and Timing Game

If the seller commits to no pre-specified bidding or allocation rule, she chooses the bid that gives her the highest expected payoff based on her beliefs regarding the type of each bidder. Because the bidding and investment involve sequential actions, the equilibrium concept for informal auctions is **Perfect Bayesian Equilibrium**. A **first-price informal auction** therefore exhibits features of a signaling and timing game of the following form:

1. Participating bidders submit informal bids simultaneously. An informal bid Π^i by bidder i is a cash payment C^i and a standard security payment $S^i(P)$.

2. The seller chooses the winning bidder rationally according to the valuation function $R(\Pi^i) = C^i + \mathbb{E}[R_\theta(S^i)|\Theta(\Pi^i)]$ provided she values the bid more than the reservation value Y. $\Theta(\Pi^i)$ is her belief of bidder i's type upon seeing the bid and all available information, and $R_\theta(S^i) = \mathbb{E}[e^{-r\tau^i_\theta} S^i(P_{\tau^i_\theta})]$, where τ^i_θ is the optimal stopping rule for type θ when bidding Π^i, that is, $\tau^i_\theta = \text{argmax}_{\tau \geq t_a} \mathbb{E}[e^{-r\tau}(P_\tau - S^i(P_\tau) - \theta)] - X - C^i$.

one-on-one negotiations, but as demonstrated in Aktas, De Bodt, and Roll (2010), even in such cases latent competition such as the threat of sale to a rival buyer is significant.

[25]DKS do not consider offer adjustments from winning and losing bidders as I do in ascending informal auctions defined later.

3. The winning bidder i pays the upfront cash C^i and the initial cost X, and then invests rationally at $\tau^i_{\theta_i}$ and makes the contingent payment.

Note the seller's valuation $R(\Pi^i)$ is not necessarily the same as the value of the security to bidder i, $C^i + R_{\theta_i}(S^i)$.[26]

We first note that if the seller does not value a bid the same as the bidder does, at least one bidder would find the seller values his security payment less, and would rather pay in cash.

Proposition 4. *An essentially unique bidding equilibrium exists for an informal FPA, which is equivalent, in terms of allocation outcome and expected payoffs, to a first-price cash auction with reserve price Y. In particular, post-auction investment is efficient.*

Note that even though an informal auction is a signaling game, the fact that cash bids are independent of the bidders' type allows us to avoid the discussion of equilibrium refinement and focus on the essentially unique equilibrium. This is also the case in DKS when cash is available.

In equilibrium, the bids are all cash-like, that is, their values are independent of beliefs on bidders' types. A better type finds it cheaper to use a security that is less sensitive to the true type and creates more social surplus. For example, using equities to separate from worse types not only inefficiently delays investment, but also costs better types more because their α shares are worth more than the worse types'. Cash-like securities ensure efficient investment and cheap separation and are thus most attractive. Moreover, because a better type is indifferent to mimicking a marginally worse type in equilibrium, all bidders must be using cash-like securities.

Given Proposition 4, and the revenue equivalence between FPAs and SPAs in cash, we see that letting bidders bid any forms of security is socially optimal.

[26] One may question if the setup of the game misses out on any informal offers, such as contracting on the timing of investment when feasible. The results are robust to additional side contracts because one can enlarge the security space from $S(P_t)$ to $S(\mathcal{I}_t)$ where $\mathcal{I}_t$ is the entire contractible information set, as long as limited liabilities hold. The proofs apply with minor changes in notations.The results also hold for non-standard security bids as long as optimal investment strategy exists.

5.2 Ascending Informal Auctions

As McAdams and Schwarz (2007) point out, in real life committing to a sealed-bid auction is hard, especially in corporate acquisitions.[27] The board of directors of a target firm has to disclose all bids to shareholders, and considers subsequent offers to avoid shareholder lawsuits. In reality, informal auctions either entail multiple rounds of negotiations and repeated communications, or manifest themselves in two-stage auctions used in privatization, takeover, and merger and acquisitions (e.g., Frankel (2011)). The former resembles an informal English auction in which buyers raise their bids until one winner emerges; Perry, Wolfstetter, and Zamir (2000) show the latter are typically robust mechanisms equivalent to an English auction. This calls for the definition of an **ascending informal auction**:

1. The seller gradually increases a numerical score R from $R = Y$, and a bidder remains in the auction if he can deliver an informal bid from a "feasible set" $\{\Pi : R(\Pi) \geq R\}$. The auction ends when only one bidder is left, and he chooses an informal bid from the final "feasible set."

2. The winning bidder i pays the upfront cash C^i and the initial cost X at t_a, and then invests rationally at $\tau^i_{\theta_i}$ and makes the contingent payment $S^i(P_{\tau^i_{\theta_i}})$, where C^i and S^i are given by his chosen final bid.

Note this variant of the English auction is equivalent to SPAs, in which bidders bid a score they generate and the winner pays the second-highest score bid.[28] This a priori is different from SPAs in which the winning bidder pays the informal bid corresponding to the second highest score. This distinction is important because the same security bids generally cost the buyers differently. In what follows, I show this ascending informal auction leads to the same expected revenue for the seller, and is not subject to renegotiation from the bidders.

[27] Even in formal auctions, such a commitment is difficult to maintain. in "Lawsuit Seeks to Block Sale of G.M. Building", New York Times, September 20, 2003, Charles Bagli documents how General Motors entertained a late offer after auctioning its Manhattan building in a first-price auction.

[28] Defining an ascending auction with multiple security bids is challenging. Closely related is Gorbenko and Malenko (2017), another study to formalize an English auction with security bids (both cash and equity).

Proposition 5. *An ascending informal auction has an essentially unique bidding equilibrium that is equivalent to a second-price cash auction with reserve price Y. Post-auction investment is efficient.*

Propositions 4 and 5 establish useful benchmark outcomes for informal bidding and post-auction investment. They do not imply bidders only use cash in informal auctions, but when bidders are not liquidity constrained, they would use contingent securities that result in the same outcomes as using cash.

In conclusion, without seller's commitment to security design, investment distortion is largely absent in informal auctions, and equilibrium bids are equivalent to cash bids. This paper therefore highlights a new advantage of using cash from the perspective of the bidders.

6 Discussion

6.1 Implications for Oil Lease Auctions

One key application of the model is the sales of natural resources, such as oil lease auctions. In many countries, the predominant design for auctioning oil leases involves fixing a royalty rate ϕ and having contractors bid up-front "bonuses" in first-price formal auctions.[29]. Because such auctions are typically modeled with interdependent values, I first extend the discussion to interdependent settings, and show that the inefficient delay in drilling is increasing and convex in volatility and royalty rate, and has positive cross partials.[30] Furthermore, the social cost of the investment lag due to the distortion associated with royalty is increasing and convex in the royalty rate ϕ. I then present corroborating evidence using data from lease sales and drilling activities in the gulf of Mexico.

[29] In the United States, the Minerals Lands Leasing Act prescribes the base share of royalty rate at 1/8 the value of production for onshore leases, and the Outer Continental Shelf Lands Act used 1/6 for offshore leases. The offshore rate for leasing beginning in 2008 is set at 18.75%. See Hendricks, Porter, and Tan (1993) and Haile, Hendricks, and Porter (2010) for more details.

[30] In addition to being analytically tractable when analyzing auction timing, the private-value framework is not unrealistic in the sense that the dispersion of bidder types over the common component, such as signals on the amount of oil reserve has decreased in recent years due to technological improvement, and the government typically provides as much information as possible to the buyers. By contrast, firms often have private drilling technologies, and retail and transportation contractors, which fit private-value settings.

"Mineral Rights" Model

Let the investment cost be $K(\theta_i, \theta_{-i})$, where K is symmetric in other bidders' report types, and has positive derivative in θ_i denoted by K_1 that is uniformly bounded by a positive constant. In the main model of this paper, $K(\theta_i, \theta_{-i}) = \theta_i$, but this specification allows other cases with interdependent values such as common-value auctions where $K(\theta_i, \theta_{-i}) = \frac{1}{N}\sum_j \theta_i$.

In "bonus-bid" auctions, the winning bidder owns a fraction $1-\phi$ of the project and has a real option value $L(\theta, \theta_{-1}) = \max_\tau \mathbb{E}[e^{-r(\tau - t_a)}((1-\phi)P_\tau - K(\theta_i, \theta_{-i}))] - X$. Scaling the cash flow in Eq.(2) gives the optimal investment threshold $P^{bonus}(\theta) = \max\{P_0, \frac{\beta}{\beta-1}\frac{K(\theta_i,\theta_{-i})}{1-\phi}\} \geq P^*(K(\theta_i, \theta_{-i}))$, thus investment is inefficiently delayed. The equilibrium bidding strategies are standard from Proposition 6.3 in Krishna (2009):

$$C(\theta) = \int_\theta^{\hat{\theta}} \frac{E[L(\theta', \theta'_{-1})|\theta'_{(1)} = \theta']}{1 - F(\theta)} f(\theta')d\theta' \tag{8}$$

where $\theta'_{(1)}$ is the type with least cost among the remaining bidders, and $\hat{\theta}$ is the cut-off type for participation.

In the proofs of Propositions 3 and 6, I have generalized Propositions 3 and 6 to interdependent-value settings. When K is such that $\theta_i \leq \theta_j$ implies $K(\theta_i, \theta_{-i}) \leq K(\theta_j, \theta_{-j})$, I show that the optimal security design is still a combination of cash and royalty payment, with the equilibrium royalty rate for type θ_i modified to

$$\phi(\theta_i, \theta_{-i}) = \frac{F(\theta_i)}{f(\theta_i)K(\theta_i, \theta_{-i}) + F(\theta_i)} \tag{9}$$

Thus a bonus-bid auction with a uniform royalty rate would not generate the highest revenue. Moreover, investments are inefficiently delayed with both bonus-bid auction and auction with optimal security design.

Using the properties of the Wald distribution, the expected inefficient time delay when a royalty rate ϕ is used is $\Gamma = -\ln(1-\phi)/[\mu - \frac{\sigma^2}{2}]$, where we have assumed $\mu - \frac{\sigma^2}{2} > 0$ for the expectation to exist.[31] Moreover, $\frac{\partial \Gamma}{\partial \phi} > 0$, $\frac{\partial \Gamma}{\partial \mu} < 0$, $\frac{\partial \Gamma}{\partial \sigma} > 0$, $\frac{\partial^2 \Gamma}{\partial \phi^2} > 0$, $\frac{\partial^2 \Gamma}{\partial \sigma^2} > 0$, $\frac{\partial^2 \Gamma}{\partial \sigma \partial \phi} > 0$.

[31] If $\mu < \sigma^2/2$, the median lag M can be considered instead.

Not only do more volatile markets or high royalty rates result in longer delays, but they are mutually reinforcing as well. What is the social cost of the investment lag? It can be shown that the option value is a fraction $(1-\phi+\phi\beta)(1-\phi)^{\beta}$ of the socially efficient value, and the fractional loss L satisfies $\frac{\partial L}{\partial \phi} > 0$, $\frac{\partial^2 L}{\partial \phi^2} > 0$. Again, royalty rate has a compounding effect on social cost.

Lease Auctions and Drilling in the Gulf of Mexico

The aforementioned predictions are consistent with available empirical evidence. The US Department of the Interior experimented with royalty auctions in 1978–1983, where the government fixed a small up-front "bonus" payment and allowed the bidders to compete on royalty rates. Many bidders bid extremely high royalty rates and the tracts were never drilled (e.g., Dougherty and Lohrenz (1980) and Binmore and Klemperer (2002)). Oil price and volatility were indeed extremely high during that period. Moreover, Humphries (2009) reports that the royalty relief programs in the 1990s significantly increased interest in deep-water leases, and oil production increased sharply. The distortion gives a potential explanation for why large tracts of land remain idle. Without prescribing detailed policy changes, this paper suggests that any useful policy recommendations should first focus on reducing the post-auction moral hazard that is inimical to both the revenue and social welfare. Moreover, instead of uniformly raising the royalty rate, allowing bidders to self select into differential rates as described in Proposition 6 could be a more effective way to increase revenue to the government, in both private-value and common-value frameworks.

To test the main message that security bids distort investment timing, we look to the Deep Water Royalty Relief Act of 1995 (DWRRA), which granted royalty relief for leases issued in the Gulf of Mexico between 1996 and 2000 at depths greater than 200 meters located wholly west of 87 degrees, 30 minutes West longitude. In appendix B. I present the institutional details, data description, and testing results. I use difference-in-difference identification strategy and find suggestive causal evidence that the lowering of royalty (mostly from 1/6 and 1/8 to zero) on average leads to 10.8% greater likelihood of exploratory drilling, consistent with the model prediction.[32] Opaluch, Grigalunas, Anderson, Trandafir, and Jin (2010) also conclude that increased royalty rates would have a net negative effect on the social value of offshore development.

6.2 Number of Bidders

Prior literature has established that in private-value auctions increasing the number of bidders enhances the seller's revenue (e.g. Bulow and Klemperer (1996)), but in our setting more bidders implies more aggressive bidding and greater moral hazard. Simulations in Figure 2 show in the spirit of Samuelson (1985) that revenue and welfare could vary in almost any way with N when we have security bids and post-auction moral hazard of investment timing. The impact of competition thus depends on the security design. Figure 3 reveals the same pattern for the case of friendly debts and call options. Since the expected social welfare and revenue to the seller need not increase with the number of potential bidders, limiting participation may improve revenue or welfare. This channel is different from the participation cost channel in Samuelson (1985) but is consistent with the observation that sellers in real life restrict the number of bidders *even absent entry fees* (see Hansen (2001) and French and McCormick (1984)).

The number of bidders also matters for security choice. Prior studies indicate that security bids usually perform better than cash bids. Rhodes-Kropf and Viswanathan (2000) show that any securities auction generates higher expected revenue to the seller than a cash auction. But since the linkage advantage of security bids lies in the extraction of the winning bidder's rent, it decreases in expectation when N increases. Yet moral hazard persists with many standard securities. In Appendix A.9, I show cash bids dominate a wide range of contingent securities in FPAs and SPAs in terms of expected revenue and social welfare, as the number of bidders gets large. The size of the bidders' market is thus an important consideration in security choice. The result also suggests security bids are rarely used when

[32] Similarly, the introduction of Area Wide Leasing in May 1983 made most of the offshore blocks of leases available for sale, including thousands of tracts in deep water areas. The royalty rates on tracts with water depth of more than 400 meters were lowered from 1/6 to 1/8. In a previous version of the paper, I use a simple Cox hazard model estimation, and also find tracts with lower royalty rate experienced greater exploratory drills.

the number of bidders is large.

6.3 Optimal Design of Auctions of Real Options

So far, other than considering various security designs, we have not given the seller full flexibility to design the auction. Conceivably, due to the post-auction moral hazard, it is possible that a seller in a formal auction may prefer not to rank the bids based on the one-dimensional index s, or to restrict the range of bidding allowed. More generally, how can the seller best design the auction?

To answer this question, I build on the insights in Board (2007b) to derive the optimal security design under the current setting, and show it can be implemented using an auction with standard security bids. The key insight lies in that the seller has to pay information rent to the bidders, but can use a bid-specific royalty rate to incentivize the winning bidder to exercise at $P^*(z(\theta_i))$, thus making the bidder face the same optimization problem. The optimal design complements this royalty rate with a corresponding bonus payment to ensure truth-telling in the bidding stage.

Proposition 6. *An optimal auction design exists and FPAs using well-ordered securities indexed by $s \in [s_L, s_H]$ implement it, where $s_H = -\underline{\theta}$, and $s_L = \max\{-\overline{\theta}, -\hat{\theta}\}$. Denote $\hat{\theta}$ as the solution to $P_0^\beta(\beta-1)^{\beta-1} = (X+Y)\beta^\beta z(\theta)^{\beta-1}$, then the securities used are of the following form:*

$$\begin{aligned} C(s) &= \frac{s}{1-\beta}\left[\frac{P_0}{P^*(z(-s))}\right]^\beta - X - \int_{s_L}^{s}\left[\frac{1-F(-s')}{1-F(-s)}\right]^{N-1}\left[\frac{P_0}{P^*(z(-s'))}\right]^\beta ds', \\ S(s,P) &= \phi(s)P, \quad \textit{where } \phi(s) = \frac{F(-s)\mathbb{1}_{\{s\in[s_L,-\underline{\theta}]\}}}{F(-s)-sf(-s)}. \end{aligned} \tag{10}$$

In equilibrium, type θ bids $s=-\theta$. Recall $P^{bonus} = \frac{\beta}{\beta-1}\frac{\theta}{1-\phi} = P^*(z(\theta_i))$, which implies that when the bidder equates his marginal benefit of waiting to his marginal cost of waiting, he and the seller face the same optimization problem. Instead of using revenue-independent strike payment $\frac{F(-s)}{f(-s)}$ for option exercises (Board (2007b)), the seller can use royalty payments to align incentives, and the delays in investments depends on the optimal auction timing, as well as the optimal security. The interpretation of the optimal security as a cash down payment plus a royalty payment relates to common discussions on security-bid auctions, and the contingent payment satisfies limited liability $S(\cdot, P) \in [0, P]$ and double monotoniticy ($S(P)$ and $P - S(P)$ being non-decreasing), as is typically required in security design.[33] Variable royalties with upfront cash are indeed frequently observed in the sales of licensing or marketing rights and contracts in publishing or movie production. These results also show that McAfee and McMillan (1987a)'s optimal linear incentive contracts are robust to time discounting, despite the fact that the discounted project payoff is actually decreasing in contractible output P.

Note that the royalty rate is increasing in θ if and only if $\frac{\theta}{z(\theta)}$ is decreasing in θ. The type with smaller actual cost relative to virtual cost thus pays less upfront cash and higher royalty rate. This result makes sense as higher royalty rate is needed to make him invest as if he bears the virtual cost. This prevents him from mimicking others, lest he pays more cash, and has a contingent residual that is more sensitive to his investment timing, which is more distorted in equilibrium. Optimal security thus involves negatively correlated cash down payments and contingent royalty payments, a novel and testable prediction that is of interest for empirical studies. That said, the optimal design does not comply with Wilson's doctrine of detail-free implementation because it requires common knowledge on bidders' type distributions.

These results relate to Myerson (1981)'s analysis in a static setting on the wedge between the seller's revenue and welfare. In addition to bidder exclusion, option exercises are inefficiently delayed under the current settings to increase revenue. Moreover, though the seller still excludes bidders, better market conditions (higher initial P_0) encourages participation and mitigates the exclusion. This implies that in real life one may not see sellers excluding bidders so much using entry fees or reserve prices because she has the alternative tool of choosing a more propitious time to hold the auction (Cong (2017)). For example, an entrepreneur selling a startup seldom excludes potential acquirers, but rather waits for the product to have higher valuations before going onto the market. Inefficiencies in dynamic

[33] See Hart and Moore (1995b), DeMarzo and Duffie (1999), and DeMarzo, Kremer, and Skrzypacz (2005).

settings are thus multi-dimensional.

6.4 General Post-Auction Moral Hazard and Security Bids

So far we have focused on the post-auction moral hazard of timing the exercise of real options. As described in the introduction, such timing issues are prevalent in corporate finance settings; oil-tract drilling, corporate expansion, and real estate developments are some salient examples. Auctions of real options therefore constitute a broad class of real-life cases. Moreover, because timing decisions are difficult and prohibitively costly to contract on, if not impossible, they represent a comprehensive and important category of post-auction moral hazard. In addition, our setup allows us to directly relate to the large literature on real options, dynamic corporate finance theory (e.g., Leland (1994)), and the theory of optimal stopping.

That said, the insight that security bids distort post-auction actions applies in greater generality. Here I discuss a general formulation of the problem and relates it to our setup and DKS's. In doing so I highlight why timing decisions are special and why our setup gives us the analytical tractability to discuss a wide range of issues.

Consider the following general setup. The net present value of the asset in a type-θ bidder's hand is $V(\theta, e)$, where e is the bidder's effort, and the effort cost is given by $c(e)$. Following DKS, we assume $V(\theta, e)$ has strict Monotone Likelihood Ratio Property with in θ, that is, fixing e, the likelihood ratio $h(V|\theta)/h(V|\theta')$ is increasing in V if $\theta < \theta'$. If $V(\theta, e)$ is deterministic, as is the case in our earlier setup, this simply implies that it is decreasing in θ. We also assume that V is weakly increasing in e, to be consistent with our earlier notations. Upon winning, a bidder who bids index s chooses effort to maximize the following,

$$\mathbb{E}[V(\theta, e) - S(V(\theta, e), s(\theta))] - c(e) \tag{11}$$

With the usual technical assumptions such as differentiability and concavity, the optimal effort can be characterized by the F.O.C.,

$$\mathbb{E}[V_e(\theta, e) - S(V_e(\theta, e), s(\theta))] = c'(e) \tag{12}$$

For a given set of well-ordered securities, denote the optimal effort for type θ with bid s as $e^*(\theta, s)$. Then following the mechanism design approach in the paper, the expected revenue to the seller is

$$N\mathbb{E}_{\vec{\theta}}\left[Q(\theta_i, \theta_{-i})\mathbb{E}\left[V(\theta_i, e^*) + F(\theta_i)/f(\theta_i)\left[\left(V_\theta(\theta_i, e^*)\right)\left(1 - S_V(V(\theta_i, e^*), e^*)\right)\right]\right]\right] \tag{13}$$

where e^* represents $e^*(\theta_i, s(\theta_i))$. In principle, various security designs can be ranked. In particular, when we take the limit that the moral hazard goes away, the problem is exactly reduced to the case in DKS. To see this, we note that for a fixed effort, a steeper security generally implies $(1 - S_V(V(\theta_i, e^*), e^*))$ is smaller, and because $V_\theta(\theta_i, e^*)$ is negative in our setting, the seller pays a smaller virtual cost — the linkage principle. Moreover, a security that gives a larger share of the surplus to the seller could distort e so much that the seller would prefer a bid that gives a smaller share but incentivize effort better.

Without making further distributional assumptions on θ and functional form assumptions on V and e, characterizing in greater detail the optimal effort, equilibrium bidding, and revenue ranking would be infeasible. Even though optimal contracts or security designs are considered in McAfee and McMillan (1987a) (and Board (2007b) for the real option case), ranking general security designs proves intractable. One attempt is Kogan and Morgan (2010) which only compares equity and debt contracts. This paper complements by considering the timing moral hazard that appears in a broad category of corporate decisions in real life, and allows general forms of security design, therefore can be viewed as an incremental step towards solving the general problem.

More specifically, we can interpret P in our baseline model as a "timing effort" e, then relabel $\beta \ln P$ as $c(e)$, $\ln(P - \theta)$ as V and $\ln \frac{P-\theta}{P-\theta-S(P,s(\theta))}$ as the security payoff, maximizing

(11) is equivalent to maximizing over P the following expression:

$$\ln(P - \theta - S(P, s(\theta))) - \beta \ln P, \tag{14}$$

which is equivalent to the optimal stopping decision the winning bidder solves, given that we have shown that the optimal stopping follows threshold strategies. So the moral hazard of timing is nested in the general formulation as well.

That said, there are several appealing features in our specification. First, $V(\theta, e)$ is deterministic in our case and the security payment depends through θ only through P and s. Removing the direct dependence of $V(\theta, e)$ on θ ensures that the linkage principle does not drive our results, allowing us to focus on the moral hazard of investment timing; second, the irreversibility of time plays a role in our setting and the moral hazard distortions can be asymmetric—a feature absent in static models, as seen in Proposition 2 and Corollaries 3 and 4; finally, absent the moral hazard distortion, a steeper security in our setting indeed corresponds to a steeper payoff in the general formulation. To see this, note that for two set of security designs S and $\hat{S}$, and an option exercise without distortion, i.e., $P = P^*(\theta)$, then

$$\begin{aligned}
\ln \frac{P - \theta}{P - \theta - S(P, s(\theta))} = \ln \frac{P - \theta}{P - \theta - \hat{S}(P, \hat{s}(\theta))} \quad &\Longleftrightarrow \quad S(P, s(\theta)) = \hat{S}(P, \hat{s}(\theta)) \\
\frac{\partial}{\partial \theta} \ln \frac{P - \theta}{P - \theta - S(P, s(\theta))} < \frac{\partial}{\partial \theta} \ln \frac{P - \theta}{P - \theta - \hat{S}(P, \hat{s}(\theta))} \quad &\Longleftrightarrow \quad \frac{\partial}{\partial \theta} S(P, s(\theta)) < \frac{\partial}{\partial \theta} \hat{S}(P, \hat{s}(\theta))
\end{aligned}$$

and the correspondence follows directly from the definition of "steepness" in DKS. These unique features and the fact that the moral hazard associated with real option exercise is prevalent imply that auctions of real options under moral hazard indeed constitute a rich and important category to warrant the detailed analysis in this paper.

7 Conclusion

Auctions of real options are ubiquitous, involve tremendous financial resources, and have policy implications. To better understand these business transactions, this paper builds on earlier studies on security-bid auctions and real options to incorporate post-auction moral hazard of option exercise. I find that both security design and seller's commitment significantly influence equilibrium outcomes. Common security designs lead to inefficient and often sub-optimal investment delays and accelerations. Contingent securities can either better or worse align the winning bidder's incentives with the seller, and security designs can be ranked using a mechanism design approach. Without sellers' commitment to security design, the bidding equilibria are equivalent to cash auctions, and post-auction investments are efficient. Taken together, the results of the paper challenge earlier approaches that analyze security-bid auctions and corporate investments separately: the interactions of these factors in dynamic settings provide a rich interplay that is not accessible otherwise, and as a consequence, many conventional understanding should be revised.

As an initial attempt to capture the salient features of auctions of real options under various settings, this paper presents a rather stylized model. More work is clearly needed. In particular, the endogenous timing of the sales is worth exploring further. Incorporating sellers' private information is also important in many applications, especially M&As. Moreover, some of the novel predictions are consistent with stylized facts, which is reassuring, and further empirical examinations may reveal more quantitative relations.

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Appendix: Derivations and Proofs

A.1 Proof of Lemma 1 and Proposition 1

Proof. Because $S(P)$ is upper semi-continuous, $P - S(P) - \theta$ of type θ upon optimal stopping is lower semi-continuous, and according to Dynkin (1963) and Dayanik and Karatzas (2003) Theorem 1.1, the value function is the smallest r-excessive majorant of $P - S(P) - \theta$ on the state space of the GBM. Because $\tilde{V} \in [-X - C, W(P_0;\theta) - X - C]$ is finite, by Proposition 5.8 and 5.10 in Harrison (2013), the optimal stopping time is first hitting. Therefore, the value function starting in a continuation region is of the following general form:

$$\tilde{V}(C, S(\cdot), \theta) = D(P_0; P_L, P_U)\left[P_L - S(P_L) - \theta\right] + D(P_0; P_U, P_L)\left[P_U - S(P_U) - \theta\right] - C - X,$$

where $D(P_0; P_L, P_U)$ is the Arrow-Debreu security that pays one dollar when P first hits lower boundary P_L before hitting upper boundary P_U, and $D(P_0; P_U, P_L)$ is similarly defined, but for first hitting P_U before hitting P_L. And $P_L \in [0, P_0]$ and $P_U \in [P_0, \infty]$ are the optimal lower and upper thresholds for investment. $D(P_0; P_L, P_U)$ satisfies $\frac{1}{2}\sigma^2 P^2 D_{PP} + \mu P D_P - rD = 0$ with the boundary conditions $D(P_0; 0, P_H) = D(P_0; P_L, \infty) = 0$, $D(P_L; P_L, P_U) = 1$ and $D(P_U; P_L, P_U) = 0$. The solution is $D(P_0; P_L, P_U) = \left(P_0^\gamma - P_0^\beta P_U^{\gamma-\beta}\right)\left(P_L^\gamma - P_L^\beta P_U^{\gamma-\beta}\right)^{-1}$, where β is given in (2) and $\gamma = 1 - 2\mu/\sigma - \beta < 0$. Similarly, $D(P_0; P_U, P_L) = \left(P_0^\beta - P_0^\gamma P_L^{\beta-\gamma}\right)\left(P_U^\beta - P_U^\gamma P_L^{\beta-\gamma}\right)^{-1}$. The optimal P_L and P_H are obviously independent of X and C and are functions of θ and P_0 in general.

Next, type θ can always do strictly better than $\tilde{\theta} > \theta$ by using $\tilde{\theta}$'s strategy, thus $\tilde{V}(C, S(\cdot), \theta)$ is decreasing in θ. For P_t in the exercise region, $\tilde{V}(C, S(\cdot), \theta) = P_t - S(P_t) - \theta - C - X$ is obviously continuous in θ. For P_t in the continuation region, consider a change of $\Delta\theta > 0$, $0 < \tilde{V}(C, S(\cdot), \theta) - \tilde{V}(C, S(\cdot), \theta + \Delta\theta) \leq \Delta\theta$ because type $\theta + \Delta\theta$ does weakly better than simply mimicking θ's strategy. As $\Delta\theta \to 0$, $\tilde{V}(C, S(\cdot), \theta + \Delta\theta) \to \tilde{V}(C, S(\cdot), \theta)$. The case of $\Delta\theta < 0$ is similar. Continuity in θ follows.

Finally, note that $\tilde{V}$ is not a function of time t, and the optimal stopping obviously involves the state variable P first hitting an upper threshold weakly greater than P, or first hitting a lower threshold weakly smaller than P.

□

A.2 Proof of Proposition 1

Proof. Double-monotonicity implies that $S(\cdot)$ is continuous, which automatically implies that $P - S(P) - \theta$ of type θ upon optimal stopping is lower semi-continuous. Note that for $P_0 > P_L$, $P_0 - S(P_0) - \theta > D(P_0; P_L, P_U)[P_L - S(P_L) - \theta]$ because $P - S(P)$ is weakly increasing in P, thus it is never optimal to start in a continuation region and wait to hit a lower boundary. I therefore conclude that starting in any continuation region, the optimal stopping is a first hitting time to an upper threshold. In other words, the optimal strategy has a payoff $\left(\frac{P_0}{P}\right)^\beta [P - S(P) - \theta]$ for threshold $P \geq P_0$. This expression is continuous, takes positive values for some $P \geq P_0$ for $\theta < \hat{\theta}$, where $\hat{\theta}$ is the cutoff type that participates and at least breaks even, and is upper-bounded by $\left(\frac{P_0}{P}\right)^\beta [P-\theta]$ which approaches 0 as $P \to \infty$. Therefore it achieves a maximum in some compact interval $[P_0, \hat{P}(\theta)]$ for some $\hat{P}(\theta) \geq P_0$, and I denote the maximum by $\tilde{P}$. The first-hitting time of $\tilde{P}$ is thus optimal among all stopping times. The proposition ensues.

□

A.3 Verification Argument for Corollary 1

Here I provide the traditional verification argument to show that the strategy given in the corollary is indeed optimal among all stopping times. This line of argument, though applicable to many commonly used securities, has to be applied case-by-case and may require additional technical assumptions. This is to be contrasted with Proposition 1 which allows us to apply FOC and SOC directly.

Since an upper threshold strategy has payoff $\left(\frac{P_0}{P}\right)^\beta [P - S(P) - \theta]$ for $P \geq P_0$, threshold $\tilde{P}$ is optimal among all upper threshold strategies. I now verify that it is optimal among all stopping times by showing the expected value following any stopping time is bounded above by the expected value associated with the $\tilde{P}$-threshold strategy.

Let $x_t = e^{-rt}\hat{W}(P_t)$, where $\hat{W}(P_t) = D(P_t;\hat{P})[\hat{P} - S(s,\hat{P}) - \theta]$ and $\hat{P} = \max\{P_t, \tilde{P}\}$. For $P \leq \tilde{P}$, using an extended version of Itô's formula (as, for example, in Karatzas and Shreve (1988), page 219), $dx_t = e^{-rt}[\mathcal{D}\hat{W}(P_t) - r\hat{W}(P_t)]dt + e^{-rt}\hat{W}_P(P_t)\sigma P_t dB_t$, where $\mathcal{D}\hat{W}(P) = \hat{W}_P(P)\mu P + \frac{1}{2}\hat{W}_{PP}(P)\sigma^2 P^2$. $\hat{W}_P$ is bounded as seen by direct computation, thus by Proposition 5B in Duffie (2009) (also found in Protter (2004)), the last term in dx_t is a martingale under the current measure. The drift is $\mathcal{D}\hat{W}(P) - r\hat{W}(P) = 0$ by the definition of β in (2). For $P > \tilde{P}$, apply Tanaka's Formula (Revuz and Yor (1999), also Karatzas and Shreve (1988)), the drift $\mathcal{D}\hat{W}(P) - r\hat{W}(P) = \mu P[1 - S'(P)] - r[P - S(P) - \theta] - \frac{1}{2}\sigma^2 P^2 S''(P) < [\mu + \frac{1}{2}(\beta - 1)\sigma^2]P[1 - S'(P)] - r[P - S(P) - \theta] < [\mu\beta + \frac{1}{2}\sigma^2\beta(\beta - 1) - r][P - S(P) - \theta] = 0$, where I have used $(1-\beta)(1 - S'(P)) = (1-\beta)(1-\alpha) < 0 = PS''(P)$ for equity, and there is no discounted local time for equity. Therefore, x_t is a super-martingale, implying for any stopping time τ, $\hat{W}(P_0) = x_0 \geq \mathbb{E}[x_\tau] = \mathbb{E}[e^{-r\tau}\hat{W}(P_\tau)] \geq \mathbb{E}[e^{-r\tau}(P_\tau - S(s, P_\tau) - \theta)]$. The equality holds for the first-hitting time with threshold $\tilde{P}$, establishing its optimality.

A.4 Proof of Proposition 2 and Corollaries 3 and 4

Proof. Note that $D(P_0; P)(P - \theta)$ is the real option value without distortion from security bids. At the optimal threshold, if the discounted security payoff can be higher by picking a higher threshold, it cannot be the case that the real option is weakly accelerated, because otherwise the real option value is also weakly higher by picking an infinitesimally higher threshold, making waiting longer a dominant deviation from the optimal threshold. The same argument applies to the case in which the discounted security payoff is weakly higher by picking a lower threshold: there is a dominant deviation to an infinitesimally lower threshold if the option value is strictly higher, unless the optimal threshold is already at P_0 which means the exercise is as efficient as in the case of cash bids.

The proofs for the corollaries are similar, but the change in the discounted security payoff is evaluated at $P^*(\theta)$. $\square$

A.5 Equilibrium Characterization for Formal Auctions

Lemma 2

Proof. For $s_1 < s_2$ and $\theta_1 < \theta_2$, because $V(s, \theta)$ is absolutely continuous with derivative in s decreasing in θ,

$$\ln\left(\frac{V(s_1,\theta_1)V(s_2,\theta_2)}{V(s_1,\theta_2)V(s_2,\theta_1)}\right) = \int_{s_1}^{s_2} \frac{\partial V(s',\theta_2)}{\partial s} ds' - \int_{s_1}^{s_2} \frac{\partial V(s',\theta_1)}{\partial s} ds' < 0 \tag{15}$$

i.e., $V(s, \theta)$ is log-submodular, and thus strictly submodular. Let $Q(s)$ be the probability of winning. Because $s(\theta) \in argmax_s Q(s)V(s,\theta) = argmax_s \ln(Q(s)V(s,\theta))$, by Topkis (1978), $s(\theta)$ is non-increasing in θ. If $s(\theta) < s_H$ were constant on an interval, the bidder with the lower θ can increase his bid marginally and increase his probability of winning (thus his payoff) by a discrete amount. Therefore $s(\theta)$ must be decreasing in type for types bidding less than s_H. Therefore, $Q(s(\theta)) = [1 - F(\theta)]^{N-1}$. Note s is also continuous in θ, lest a type right below a discontinuity could lower his bid marginally without affecting the chance of winning.

Next, by direct revelation, $\theta \in \text{argmax}_{\theta' \in [\underline{\theta},\overline{\theta}]} Q(s(\theta'))V(s(\theta'), \theta)$. For any $\theta' < \theta$,

$$Q(s(\theta))V(s(\theta),\theta) \geq Q(s(\theta'))V(s(\theta'),\theta) = Q(s(\theta'))[V(s(\theta),\theta) + V_1(s^*,\theta)[s(\theta') - s(\theta)]]$$

for some s^* between $s(\theta')$ and $s(\theta)$. Since $V_1 < 0$, the above expression can be written as

$$\frac{Q(s(\theta')) - Q(s(\theta))}{\theta' - \theta} \frac{V(s(\theta),\theta)}{-Q(s(\theta'))V_1(s^*,\theta)} \geq \frac{s(\theta') - s(\theta)}{\theta' - \theta}$$

Similarly, exchanging θ and θ', for some s^{**} between $s(\theta)$ and $s(\theta')$,

$$\frac{Q(s(\theta')) - Q(s(\theta))}{\theta' - \theta} \frac{V(s(\theta'),\theta')}{-Q(s(\theta))V_1(s^{**},\theta')} \leq \frac{s(\theta') - s(\theta)}{\theta' - \theta}$$

Taking the limit we get (5).

As $V(s, \theta)$ is continuous in s over $[s_L, s_H]$ and decreasing in θ, $\max_s V(s, \theta)$ exists and $\sup\{\theta \leq \overline{\theta} | [1 - F(\hat{\theta})]^{N-1} \max_s V(s, \hat{\theta}) \geq 0\}$ gives the cutoff type. In equilibrium, $s(\hat{\theta}) = \sup\{s \in [s_L, s_H] | V(s, \hat{\theta}) \geq 0\}$, otherwise bidding slightly more increases the winning probability discretely from zero while still breaking even upon winning. As $V(s(\hat{\theta}), \hat{\theta}) \leq W(P_0; \hat{\theta}) - X$ and $W(P_0; \hat{\theta}) - X = 0$ in cash auctions, the cutoff type for security bids is in general weakly smaller than that in cash auctions. With the absolute continuity assumption in the proposition, the cutoffs are the same as in cash auctions.

This establishes uniqueness of the equilibrium, whose existence follows from the sufficiency of bidders' F.O.C. - the quasiconcavity of $\ln(Q(s)V(s, \theta))$. For any $s' \in (s(0), s(\theta))$, $\exists \theta' \in (0, \theta)$ such that $s(\theta') = s'$. Submodularity of V implies $\frac{\partial}{\partial s} \ln[Q(s')V(s', \theta)] > \frac{\partial}{\partial s} \ln[Q(s')V(s', \theta')] = 0$. Similarly, $\frac{\partial}{\partial s} \ln[Q(s')V(s', \theta)] < 0$ for $s' \in (s(\theta), s(\underline{\theta}))$. Therefore for every θ, there exists a unique s maximizing $Q(s)V(s, \theta)$. $\square$

Lemma 3

Proof. Since Π is a left-continuous map, $V(s,\theta)$ is left-continuous in s by an argument similar to the one in Lemma 1 for $V(s,\theta)$ to be continuous in θ. Therefore $s(\theta)$ is well-defined. Suppose a participating bidder of type θ bids $s > s(\theta)$, he benefits from decreasing s to reduce the states of the world in which he wins but receives negative payoff. Similarly, he wants to increase s when $s < s(\theta)$, assuming any indifference in bidding is resolved by bidding higher. As $V(s,\theta)$ is decreasing in θ, for $\theta' > \theta$, $V(s(\theta),\theta') < V(s(\theta),\theta) = 0 = V(s(\theta'),\theta')$. Thus $s(\theta) > s(\theta')$, leading to $s(\theta)$ being decreasing. The cut-off type is the same as in FPAs by an argument similar to that in the proof of Lemma 2. □

Corollary 5

Proof. When P_0 increases or X decreases, $\hat{\theta}$ weakly increases, thus potentially a positive measure of originally non-participating bidders are bidding. Given the bidding strategy in SPAs and the fact that non-negative V is non-increasing in s, decreasing in X and increasing in P_0, for each original participant increasing P_0 or decreasing X results in a bigger s. In FPAs, let $\tilde{s}(\theta)$ denote the bidding strategy after P_0 increases or X decreases. Then $\tilde{s}(\theta) \geq s(\theta)$ at least for the original cutoff type $\hat{\theta}_{old}$. If $\tilde{s}(\theta) = s(\theta)$ for any $\theta \in [\underline{\theta}, \hat{\theta}_{old}]$, $\tilde{s}'(\theta) > s'(\theta)$ by Lemma 2, thus $\tilde{s}(\theta)$ stays above $s(\theta)$ for a positive measure of types. Overall we have a positive measure of types bidding bigger s, and all types bidding weakly bigger s. For the same reason, the result holds when N increases in FPAs. Consequently, the winner bids a greater index with more competition, smaller initial cost, or higher threshold for auction. □

A.6 Proof of Proposition 3

Proof. Let $Q(\tilde{\theta}_i, \theta_{-i})$ be the probability of allocating the project to bidder i, who has investment cost $K(\theta_i, \theta_{-i})$, where K is symmetric in other bidders' report types, and has positive derivative in θ_i denoted by K_1 that is uniformly bounded by a constant $A > 0$. In the main model of this paper, $K(\theta_i, \theta_{-i}) = \theta_i$, but this specification allows other cases with interdependent values such as common-value auctions.

The expected utility at time zero to type θ_i upon participating and optimally investing is

$$U(\theta_i, \tilde{\theta}_i) = \mathbb{E}_{\theta_{-i}}\left[Q(\tilde{\theta}_i, \theta_{-i}) \max_{\tau \geq t_a} \mathbb{E}_P \left[e^{-r\tau}(P_\tau - K(\theta_i, \theta_{-i})) - \int_{t_a}^{\infty} e^{-rt} S(\tilde{\theta}_i, \theta_{-i}, \mathcal{I}_t) dt - e^{-rt_a} X\right]\right].$$

As $S(\tilde{\theta}_i, \theta_{-i}, \mathcal{I}_t)$ could be artificially constructed that an optimal stopping time for exercising the real option may not exist, it is reasonable to focus attention on the set of $S(\tilde{\theta}_i, \theta_{-i}, \mathcal{I}_t)$ such that an optimal stopping time exists for all types under a direct mechanism. With this restriction, let $\tau^*(\theta_i, \tilde{\theta}_i, \theta_{-i})$ denote the optimal stopping time that is almost surely bigger than t_a, and $\tau_i^* = \tau^*(\theta_i, \theta_i, \theta_{-i})$. Incentive compatibility requires $U(\theta_i) \equiv U(\theta_i, \theta_i) \geq U(\theta_i, \tilde{\theta}_i)$ and the individual rationality requires $U(\theta_i) \geq 0$.

The IC constraint can be written as $\theta_i \in \text{argmax}_{\tilde{\theta}_i \in [\underline{\theta}, \overline{\theta}]} U(\theta_i, \tilde{\theta}_i)\ \forall\ i$. Let $a = (\tau, \tilde{\theta})$ denote the action pair of reporting $\tilde{\theta}$ and rationally exercise following the stopping time τ. Let

$$g(a,\theta) = Q(\tilde{\theta},\theta_{-i})\mathbb{E}_P\left[e^{-r\tau}(P_\tau - K(\theta_i,\theta_{-i})) - \int_{t_a}^{\infty} e^{-rt}S(\tilde{\theta},\theta_{-i},\mathcal{I}_t)dt - e^{-rt_a}X\right]$$

Then following the argument in Milgrom and Segal (2002), for any $\theta',\theta'' \in [\underline{\theta},\overline{\theta}]$ with $\theta' < \theta''$,

$$\begin{aligned}
|U(\theta') - U(\theta'')| =& \mathbb{E}_{\theta_{-i}}\left[\left|\sup_{a'} g(a',\theta') - \sup_{a''} g(a'',\theta'')\right|\right] \\
\leq& \mathbb{E}_{\theta_{-i}}\left[\sup_a |g(a,\theta') - g(a,\theta'')|\right] = \mathbb{E}_{\theta_{-i}}\left[\sup_a \left|\int_{\theta''}^{\theta'} g_\theta(a,\theta)d\theta\right|\right] \\
\leq& \mathbb{E}_{\theta_{-i}}\left[\int_{\theta'}^{\theta''} \sup_a |g_\theta(a,\theta)|\, d\theta\right] \leq A|\theta'' - \theta'|
\end{aligned}$$

This implies $U(\theta)$ is absolutely continuous, and thus differentiable everywhere. $U(\theta) = U(\hat{\theta}) - \int_\theta^{\overline{\theta}} U'(\theta')d\theta'$. By Theorem 1 in Milgrom and Segal (2002), $U'(\theta) = g_\theta(a^*,\theta)$. Writing it in the integral form gives that any incentive compatible and individually rational mechanism satisfies

$$U(\theta_i) = \mathbb{E}_{\theta_{-i}}\left[\int_{\theta_i}^{\overline{\theta}} Q(\theta_j,\theta_{-i})\mathbb{E}_P[e^{-r\tau_j^*}]K_1(\theta_j,\theta_{-i})d\theta_j\right] + U(\overline{\theta}) \tag{16}$$

where $U(\overline{\theta}) \geq 0$. Moreover $\tau_i \geq t_a$ $\forall i$ for time consistency.

The ex-ante social welfare is $N\mathbb{E}_{\vec{\theta}}[Q(\theta_i,\theta_{-i})\left(\mathbb{E}_P[e^{-r\tau_i^*}(P_{\tau_i^*} - K(\theta_i,\theta_{-i}))] - e^{-rt_a}X\right)]$, and the seller's *ex ante* revenue is the social welfare less the agents' ex-ante utilities: $N\mathbb{E}_{\vec{\theta}}[Q(\theta_i,\theta_{-i})(\mathbb{E}_P[e^{-r\tau_i^*}(P_{\tau_i^*} - K(\theta_i,\theta_{-i}))] - e^{-rt_a}X)] - N\mathbb{E}_{\vec{\theta}}[U(\theta_i)]$. Using (16) and taking expectations over the winning bidder's type, it becomes

$$N\mathbb{E}_{\vec{\theta}}[Q(\theta_i,\theta_{-i})(\mathbb{E}_P[e^{-r\tau_i^*}(P_{\tau_i^*} - K(\theta_i,\theta_{-i}) - F(\theta_i)/f(\theta_i)K_1(\theta_i,\theta_{-i}))] - e^{-rt_a}X)] - NU(\overline{\theta}) \tag{17}$$

When $K(\theta_i,\theta_{-i}) = \theta_i$, this simplifies to $N\mathbb{E}_{\vec{\theta}}[Q(\theta_i,\theta_{-i})(\mathbb{E}_P[e^{-r\tau_i^*}(P_{\tau_i^*} - z(\theta_i))] - e^{-rt_a}X)] - NU(\overline{\theta})$. With standard securities, a participant with the least cost wins, the proposition follows. □

A.7 Proof of Proposition 4

Proof.

Lemma 4. *The seller and a participating bidder i have the same valuation for bid Π^i, that is, $R(\Pi^i) = C^i + R_{\theta_i}(S^i)$.*

Proof. If this were not the case, at least one bidder would find the seller values his security payment less than he does, and would rather pay the seller's valuation in cash, which keeps his marginal probability of winning the same. Formally, the lemma is obviously true if only one type uses Π^i. If more than one type use this bid, either it holds or one of the types θ_1 has $R_{\theta_1}(S^i) + C^i \neq R(\Pi^i)$. Then $\exists$ θ_2 (potentially $= \theta_1$) s.t. $R(\Pi^i) < C^i + R_{\theta_2}(S^i)$. Consider the deviation for bidder 2 in the subgame to a cash bid equal to $R(\Pi^i)$

and invest efficiently. This deviation is profitable because he creates weakly greater social surplus, pays less, and has the same marginal probability of winning. Thus by contradiction $R(\Pi^i) = C^i + R_{\theta_i}(S^i)$ always. $\square$

This in turn leads to:

Lemma 5. *In a bidding equilibrium, a participating bidder i has $\tau^i_{\theta_i} = \tau^*_i$, where τ^*_i is the stopping time corresponding to the threshold strategy with investment trigger $P^*(\theta_i)$.*

Proof. The intuition is that if a bidder does not invest efficiently upon winning, he can always deviate to a bid that results in efficient investment, and offer more cash to the seller to increase his marginal probability of winning without reducing the payoff upon winning.

Now suppose $\tau^i_{\theta_i} \neq \tau^*_i$, consider deviating to a cash bid $C = R(\Pi^i)$. The payoff from deviation $\mathbb{E}[e^{-r\tau^*_i}(P_{\tau^*_i} - \theta_i)] - R(\Pi^i)$ dominates the original payoff $\mathbb{E}[e^{-r\tau^i_{\theta_i}}(P_{\tau^i_{\theta_i}} - \theta_i - S^i(P_{\tau^i_{\theta_i}}))] - C^i = \mathbb{E}[e^{-r\tau^i_{\theta_i}}(P_{\tau^i_{\theta_i}} - \theta_i)] - R_{\theta_i}(S^i) - C^i$. Thus the deviation is profitable and the claim follows. $\square$

The intuition is that if a bidder does not invest efficiently upon winning, he can always deviate to a bid that results in efficient investment, and offer more cash to the seller to increase his marginal probability of winning without reducing the payoff upon winning.

Lemma 6. *Informal auctions only admit fully-separating equilibria.*

Proof. Suppose a non-singleton set Θ_p of types pool to bid Π in FPAs, or have the same drop-out bid in SPAs. The claim follows if there is always a profitable deviation by a type in this set.

From Lemma 5, a type θ in expectation pays $C + D(P_0; P^*(\theta))S(P^*(\theta))$. Let $\theta_k = \operatorname{argmax}_{\theta' \in \Theta_p} R_{\theta'}(S)$ where $R_{\theta'}(S) = D(P_0; P^*(\theta'))S(P^*(\theta'))$. Then $R(\Pi) \leq C + D(P_0; P^*(\theta_k))S(P^*(\theta_k))$. If the inequality is strict, type k can profitably deviate to cash bid $R(\Pi)$. Otherwise, $R_{\theta_i}(S) = R_{\theta_j}(S) = R(\Pi) - C$, for some $\theta_i < \theta_j$ both in Θ_p, but there is still a profitable deviation:

I first argue that Θ_p contains a positive measure of types. For any $\theta_n \in (\theta_i, \theta_j) \bigcap \Theta^c_p$, call his bid $\tilde{\Pi}$. Let Q and $\tilde{Q}$ be the probability of winning when bidding Π and $\tilde{\Pi}$. Since θ_i does not want to deviate to cash bid $R(\tilde{\Pi})$, $Q[W(P_0; \theta_i) - R(\Pi) - X] \geq \tilde{Q}[W(P_0; \theta_i) - R(\tilde{\Pi}) - X]$. Similarly, $Q[W(P_0; \theta_j) - R(\Pi) - X] \geq \tilde{Q}[W(P_0; \theta_j) - R(\tilde{\Pi}) - X]$. As $\theta_i \neq \theta_j$, the equality signs cannot hold simultaneously. Thus for $\theta_n \in (\theta_i, \theta_j)$, $Q[W(P_0; \theta_n) - R(\Pi) - X] > \tilde{Q}[W(P_0; \theta_n) - R(\tilde{\Pi}) - X]$. This means θ_n can profitably deviate to cash bid $R(\Pi)$. Therefore, it has to be that $[\theta_i, \theta_j] \in \Theta_p$.

Next, note $W(P_0; \theta_i) - X - R_{\theta_i}(S) - C > W(P_0; \theta_j) - X - R_{\theta_j}(S) - C \geq 0$. Type θ_i can deviate profitably to cash bid $\epsilon + R(\Pi)$ which reduces his payoff by ϵ upon winning but increases his marginal chance of winning by a discrete amount (because he separates from a positive measure of types). $\square$

As every bidder upon winning invests efficiently, a better type has greater valuation than worse types and can separate. This also implies that no two bidders place the same bid.

With these lemmas, the equilibrium bids turn out to be rather simple: because every bidder upon winning invests efficiently, a better type generates greater social surplus and can offer more to separate from worse types. This lemma also implies that no two bidders place the same bid.

With the lemmas, now consider the bidding strategy from a FPA in cash. The valuations for the bids are simply the cash amounts. I show there exists a belief that supports an equilibrium with this bidding strategy in the informal auction. First, there would not be any deviation to another cash amount since the bidding strategy comes from the equilibrium in FPA cash auction. Next, for beliefs such that upon seeing an out-of-equilibrium bid Π^i, the auctioneer believes it comes from $\tilde{\theta}_i = \text{argmin}_{\theta \in [\underline{\theta}, \overline{\theta}]}[R_\theta(S^i) + C^i]$ and gives it a valuation $\tilde{R}$. If bidder i finds this deviation attractive (yielding an expected payoff more than the original amount after cash payments), then he also finds deviating to cash bid $\tilde{R}$ weakly more attractive, contradicting the fact that no deviation to another cash amount is profitable. Thus the equilibrium from a first-price cash auction is an equilibrium in the informal auction. The argument also applies to cash-like bid Π such that $R(\Pi)$ is independent of the seller's belief on the bidders' types.

Next I show any bidding equilibrium in the informal auction has the same allocation outcome and expected payoffs as cash auctions. The seller forms correct beliefs about types since Lemma 6 rules out pooling. Bidder i's bid S^i can be replaced by an equivalent cash bid. This would not change the marginal probability of winning by Lemma 4, neither does it change the payoff upon winning as Lemma 5 implies the total surplus is the same. Since the bidders face the same maximization problem as in a FPA with cash, almost every bid is cash-like in terms of its expected payoff. □

A.8 Proof of Proposition 5

Proof. Lemma 4 applies to the winner's final bid and the losers' drop-out bids, and since bidder i can bid a maximum value of $W(P_0; \theta_i) - X$, bidding until the score reaches this value is an undominated strategy. This implies Lemma 6 is true for ascending informal auctions. Finally, Lemma 5 applies to the winner's bid; otherwise, a profitable deviation using cash exists.

In equilibrium, the proof of Lemma 4 goes through for the winner's final bid and each type's drop-out bids, otherwise there must be multiple types using the same bid and for some type, its bid is undervalued and he can profitably deviate to bidding cash. Since by bidding cash, bidder i has a value of $W(P_0; \theta_i) - X$, so he would remain in the game before the score surpasses this value. Since $W(P_0; \theta_i) - X \neq W(P_0; \theta_j) - X$ for $\theta_i \neq \theta_j$, different types drop out at different score values, resulting in a separating equilibrium strategy of dropping out. When there is one bidder remaining, he has to pay a score at which the second last bidder drops out. If the bid does not lead to efficient investment, the winning bidder can simply bid cash equal to the score, and increase his own profit by investing efficiently. Therefore Lemma 5 holds.

With these results, the allocations, payoffs, and investment outcomes in a bidding equilibrium are identical to those in an ascending second price auction in cash. By revenue equivalence theorem, they are equivalent to those in first-price and second-price cash auctions too. The auction timing strategy by the seller is thus the same as that in a first-price informal auction. □

A.9 Discussion of Cash Dominating M-regular Securities

Proof. Suppose $\underline{s}$ is the security the type $\underline{\theta}$ bids, without loss of generality, $b_i(\underline{s}) \leq b_j(\underline{s})$ if $i \leq j$. I define ***M-regular security*** to be a class of contingent securities in the form $\sum_{i \in I} a_i(s)[P - b_i(s)]^+$, where I is a countable set and $\sum_i a_i(s) \leq 1 \ \forall s$, such that for $M > 0$, and $\frac{\beta}{\beta-1}\underline{\theta} \in [b_m, b_{m+1})$,

$$\min\left(\left|\frac{\beta}{\beta-1}\underline{\theta} - b_m\right|, \left|\frac{\beta}{\beta-1}\underline{\theta} - b_{m+1}\right|, \left|\sum_{i \leq m} a_i b_i - \underline{\theta}\sum_{i \leq m} a_i\right|\right) > M. \tag{18}$$

Most common securities are M-regular securities or can be closely approximated by M-regular securities. For example, equity corresponds to $a_1 = \alpha(\underline{\theta})$, $a_2 = b_1 = 0$, $b_2 = \infty$. For any $M > 0$, cash bids dominate M-regular securities in FPAs and SPAs in terms of expected revenue and social welfare, as the number of bidders gets large.

To show this, consider pure contingent securities. Extension to include cash is straightforward. Conditional on an auction timing, cash auctions lead to efficient investments and obviously dominate in terms of welfare. For the seller's revenue, first consider SPAs. The revenue is $\mathbb{E}[e^{-r\tilde{\tau}} S(s(\theta_{(2)}), P_{\tilde{\tau}}) \mathbb{1}_{\{\theta_{(2)} \leq \hat{\theta}\}}]$ $= \mathbb{E}[\left(e^{-r\tilde{\tau}}(P_{\tilde{\tau}} - \theta_{(1)}) - U(\tilde{\tau}, s(\theta_{(2)}), \theta_{(1)})\right) \mathbb{1}_{\{\theta_{(2)} \leq \hat{\theta}\}}] \leq \mathbb{E}[\left(e^{-r\tilde{\tau}}(P_{\tilde{\tau}} - \theta_{(1)}) - X\right) \mathbb{1}_{\{\theta_{(1)} \leq \hat{\theta}\}}] \equiv R_0$, where $\tilde{\tau} = argmax_\tau U(\tau, s(\theta_{(2)}), \theta_{(1)})$ and $U(\tau, s, \theta) = \mathbb{E}[e^{-r\tau}(P_\tau - S(s, P_\tau) - \theta)]$. Similarly in FPAs, the revenue is bounded above by R_0 with $\tilde{\tau} = argmax_\tau U(\tau, s(\theta_{(1)}), \theta_{(1)})$. Let s_w denote the index the winning bidder pays in general. Then in FPAs and SPAs, the revenue is bounded above by R_0 with $\tilde{\tau} = argmax_\tau U(\tau, s_w, \theta_{(1)})$

The revenue from cash auction would be the expected second highest valuation $R_2 \equiv \mathbb{E}[(W(P_0; \theta_{(2)}) - X)\mathbb{1}_{\{\theta_{(2)} \leq \hat{\theta}\}}]$. When $N \to \infty$, $\theta_{(2)} - \theta_{(1)} \overset{a.s.}{\to} 0$. Thus $W(P_0; \theta_{(2)}) - W(P_0; \theta_{(1)}) \overset{a.s.}{\to} 0$. Now $\mathbb{1}_{\{\theta_{(2)} \leq \hat{\theta}\}}$ and the above are bounded, by bounded convergence, R_2 converges a.s. to $R_1 \equiv \mathbb{E}[(W(P_0; \theta_{(1)}) - X)\mathbb{1}_{\{\theta_{(1)} \leq \hat{\theta}\}}]$.

If $R_1 - R_0$ converges to a quantity bounded below by a positive constant, the claims follow. First note $U(\tau, s_w, \theta_{(1)})$ admits an optimal stopping solution involving threshold strategies. To see this, write $U(\tau, s_w, \theta_{(1)}) = D(P_0; P)[P - \theta_{(1)} - \sum_{i \in I} a_i(s_w)[P - b_i(s_w)]^+]$, which admits a maximizer $\tilde{P}(\theta_{(1)})$. Then use that as an investment trigger and apply the standard verification argument. Next, as $\theta_{(1)} - \underline{\theta} \overset{a.s.}{\to} 0$, the investment trigger in cash auctions converges to $P^* = \frac{\beta}{\beta-1}\underline{\theta}$, and $\tilde{P}(\theta_{(1)})$ to $\tilde{P}^* = \tilde{P}(\underline{\theta})$. Whether $\tilde{P}^* \in [b_m, b_{m+1})$ or not, $|\tilde{P}^* - P^*| \geq M$. Since P^* is the optimal trigger for $\mathbb{E}[e^{-r\tau}(P_\tau - \underline{\theta})]$, $R_1 - R_0 \overset{a.s.}{\to} \epsilon$ for some $\epsilon > f(M)$, where $f(M)$ is a function of M that is positive and independent of N. Therefore as N becomes big, R_2 converges to R_1 which dominates R_0 in the limit. Thus cash auctions yield higher revenue than the security-bid auctions. □

A.10 Proof of Proposition 6

Proof. To maximize seller's revenue, for every realization of the types and any allocation rule, the seller wants winner θ_i to invest when P first hits $P^*(z(\theta_i))$. The proposed contingent payment achieves this outcome because given the royalty rate for type θ, the investment threshold is $P^{bonus} = \frac{\beta}{\beta-1}\frac{\theta}{1-\phi} = \frac{\beta}{\beta-1}z(\theta)$. Moreover, $U(\overline{\theta}) = 0$ and the project is only allocated to types that contribute positively to the revenue. z is increasing in θ leads to the unique cutoff type $\hat{\theta}$ proposed and allocation to a participant with the smallest θ. With interdependent values, for the same set of realized types, assume $K(\theta_i, \theta_{-i}) \leq K(\theta_j, \theta_{-j})$ if $\theta_i \leq \theta_j$. Then the cutoff type is well defined and the type with the smallest θ gets allocated the real option, if at all.

That $U(\theta_i)$ is decreasing in θ_i implies any mechanism satisfying the above meets IR of all types. Suppose $\theta_i < \tilde{\theta}_i$, (16) leads to $U(\theta_i, \tilde{\theta}_i) = U(\tilde{\theta}_i) - \int_{\theta_i}^{\tilde{\theta}_i} U_1(\theta, \tilde{\theta}_i, \tau^*(\theta, z_i(\tilde{\theta}_i, \theta_{-i}, \cdot)))d\theta \leq U(\tilde{\theta}_i) - \int_{\theta_i}^{\tilde{\theta}_i} U_1(\theta, \theta, \tau^*(\theta, z_i(\theta, \theta_{-i}, \cdot)))d\theta$ $= U(\theta_i)$, where the inequality follows from the differential form of (16) and the fact that reporting a higher investment cost leads to a lower probability of winning and a later investment. Similarly, $U(\theta_i, \tilde{\theta}_i) \leq U(\theta_i)$ for $\theta_i > \tilde{\theta}_i$. Thus incentive compatibility holds if (16) holds, which requires the $C(\theta_i, \theta_{-i})$ given in the proposition. □

B. Institutional Details and Empirical Evidence

Oil and Gas Auction and Drilling in the Gulf of Mexico

Offshore drilling activities in the gulf of Mexico date back to the 1940s. The US Congress passed the Outer Continental Shelf Lands Act (OCSLA) in 1953 to grant the Department of the Interior the authority for conducting lease auctions, collecting royalties, and overseeing all activities associated with the drilling in federal waters. The Minerals Management Service (MMS) traditionally conducted the lease auctions, but due to a reorganization in response to the DeepWater Horizon oil spill in 2010, it was replaced by the Bureau of Ocean Energy Management (BOEM) and the Bureau of Safety and Environmental Enforcement (BSEE). Most leases were sold in "Bonus-bid" auctions, where the royalty rate on future revenue is fixed and the bidders bid upfront cash. The current royalty rate is standardized at 18.75%, but has historically taken on different values at various times and for different leases.

The empirical tests in this paper utilize the following policy change: Common incentives for offshore oil and gas development include certain forms of royalty relief. The OCSLA authorized the Secretary of Interior to grant royalty relief to promote increased production in oil and gas (43 U.S.C. 1337). The Deep Water Royalty Relief Act of 1995 (DWRRA) expanded the Secretary's royalty relief authority in the Gulf of Mexico outer continental shelf. Eligible leases are those issued in the Gulf of Mexico between 1996 and 2000 at depths greater than 200 meters located wholly west of 87 degrees, 30 minutes West longitude. Interest in deep water surged after the enactment of the act (August 8, 1995), with 3,000 deepwater leases bid between 1996 and 1999. There is also significant increase in annual deepwater oil production. Greater details on various royalty relief programs can be found at BOEM's official website: http://www.boem.gov/Oil-and-Gas-Energy-Program/Energy-Economics/Royalty-Relief/Index.aspx.

Data Construction

Data on the lease auctions, drilling, and mineral production are from the Minerals Management Service of the Department of Interior. I observe detailed lease-level variables, bolehole-level variables, lease sale data, ownership data, and production data. For the cost of drilling and equipping a borehole that varies by year, region, well type, and well depth, I use John Beshears's inflation-adjusted estimates based on annual surveys by the American Petroleum Institute (API) and GDP implicit price deflator index from the Bureau of Economic Analysis. The detailed description of various variables are in Beshears (2013). The estimation uses leases auctioned in 1991-2000 with a total of 7858 leases.

Monthly prices for oil and natural gas are obtained from the World Bank Commodity Price Data (Index Mundi Data Set and the Energy Information Administration (EIA) also contain futures prices, but are not monthly). The prices are inflation-adjusted using monthly CPI data from the Federal Reserve Bank in St. Louis. ($http://research.stlouisfed.org/fred2/graph/?s[1][id] = CPIAUCSL$). The discussion in this paper uses average spot prices and the results are robust to using different categories of crude oil and natural gas (such as West Texas Intermediate, Brent Crude Oil, etc.).

The key variables in the empirical exercises are listed below:

Event: The event is first exploratory drill. For this, I take the spud date for the first exploratory bolehole drilled in each lease tract.

DEEP: Dummy for leases with water depth greater than 200 meters in areas west of the 87 degrees 30 minutes West longitudinal line.

RELIEF: Dummy for the implementation of DWRRA. It takes the value 0 for leases auctioned before August 1995 and 1 afterward.

DEEP*RELIEF: Interaction term for the DEEP and RELIEF dummies. The coefficient for this variable is of interests in the diff-in-diff test.

RTY: The royalty rate specified in the lease agreement. It is typically 1/6 or 1/8 in this data set.

DUR: The number of days from lease auction to expiration. Most lease terms are 5-10 years.

DEPTH: The water depth of the leased tract. The results reported use minimum water depth, and are robust to alternative specifications using maximum water depth or average water depth.

SIZE: The area covered under each lease. Most leases had an area of approximately 5,000 acres, though some were smaller.

MKT: The number of leases sold in the same sale as a proxy for the market demand for oil and gas leases. When the market demand is high, the quality of the marginal lease sold may be low in the sense that the reserve quantity is small or there is huge uncertainty, which in turn affects drilling decisions.

P(t): Average spot price for oil and gas. The results are robust to the inclusion of oil and gas prices separately, or prices lagged by 1-5 months.

VOL(t): Average trailing-12-month volatilities for oil and gas spot prices. The results are robust to the inclusion of oil and gas price volatilities separately and to variations within one year of the trailing window.

COST(t): The industry average drilling cost for dry, oil, and gas boreholes. The drilling cost is the initial cost plus the equipping cost.

Empirical Models and Tests

I employ a Cox proportional hazard model to test whether a lower royalty rate leads to greater likelihood of exploratory drill. For identification, I use difference-in-difference method. Cox hazard models probably represent the state of the art in survival analysis with reduced-form models. They make the assumption that the hazard rate $\kappa(t)$ of exploratory drill at time t conditional on lack of drill until time t is $\kappa(t) = \psi(t)[exp(\vec{X}(t)^T\vec{\gamma})]$, where $\psi(t)$ is the baseline hazard rate that is completely unrestricted, and $X(t)$ is a vector of independent variables listed earlier (AWL is only for the first hypothesis where as *DEEP*, *RELIEF*, *DEEP*RELIEF* are only for the second). This specification handles censoring of observations and allows time-varying covariates. There is no survivorship bias or response bias because the leases are sampled at birth (the auction), and all leases are recorded by the Department of Interior.

Table 2 gives one analysis using leases auctioned in 1991-2000. The coefficient of $DEEP * RELIEF$ indicates that the implementation of DWRRA led to an increase in the difference in exploration likelihood between treated and untreated groups.

Tables and Figures

Table 2: Exploration of Oil and Gas Tracts

This table presents estimates from a Cox regression with time-varying covariates. The dependent variable is time-to-exploratory-drill, which measures the number of days from the lease auction to the first exploration. The independent variables are *DEEP* indicating the DWRRA-treated group, *RELIEF* indicating whether DWRRA is implemented, the variable of interest *DEEP*RELIEF* capturing the interaction, oil and gas price measure $P(t)$, price volatility $VOL(t)$, drilling cost $COST(t)$, royalty rate *RTY*, water depth *DEPTH*, lease length *DUR*, tract size *SIZE*, market demand *MKT*, firm fixed effects *FIRM f.e.*, and information externality *INFO*. If the dependent variable is observed without any realization, it is treated as a censored event. Model χ^2 reports the joint significance of the estimates.

Variables	Hazard Ratio	Coefficient	Standard Error
*DEEP*RELIEF*	1.1080***	0.1024***	0.0689
DEEP	1.0830	0.0794	0.1337
RELIEF	0.5389***	-0.6183***	0.1958
MKT	0.9989	-0.0011***	0.0002
DEPTH	0.9997***	-0.0003***	0.0001
SIZE	1.0000**	0.0000**	0.0000
RTY	1.0120	0.0117	0.0517
DUR	1.0040	0.0039	0.0047
P(t)	1.0790***	0.0764***	0.0277
VOL(t)	1.0150	0.0152	0.0172
COST(t)	0.9996	-0.0004	0.0010
No. of leases = 7858	Model $\chi^2 = 451$	*FIRM f.e.* Yes	*INFO* Yes

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

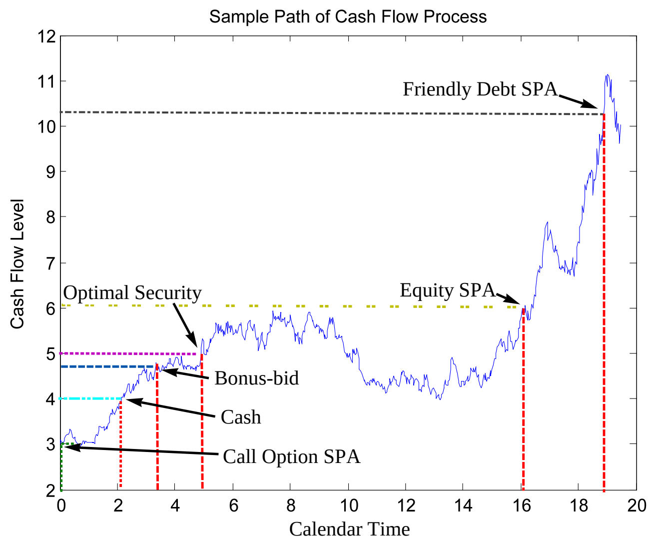
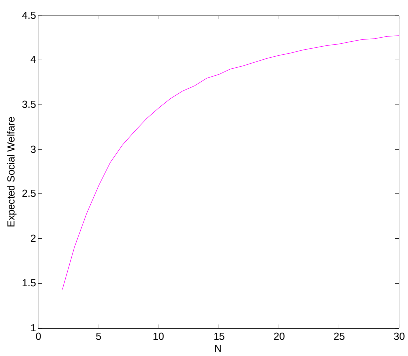
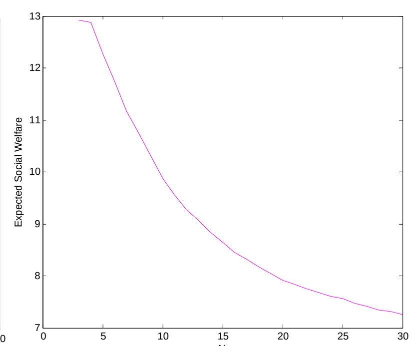


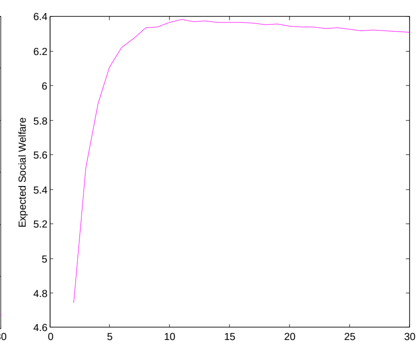
Figure 1: Investment thresholds under various security designs. Simulated with $\mu = 0.06$, $\sigma = 0.2$, $r = 0.16$, $\theta \sim Unif[1.5, 5]$, $X = 0.4$, $Y = 0$, $P_0 = 3$.



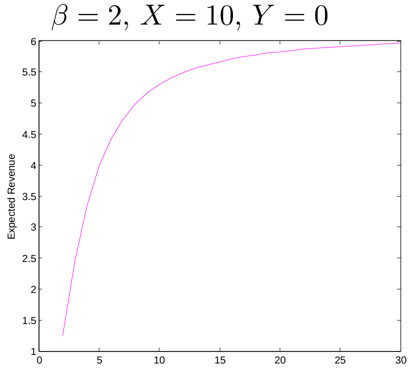
(a) Unif$[20, 50]$, $P_0 = 35$, $\beta = 2$, $X = 10$, $Y = 0$



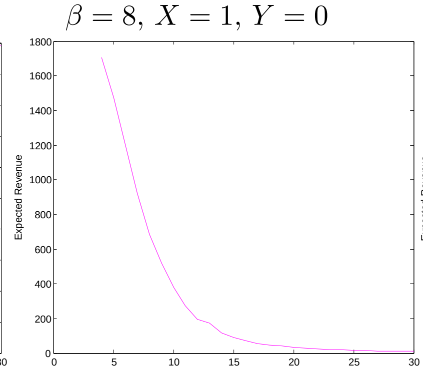
(b) Unif$[20, 50]$, $P_0 = 35$, $\beta = 8$, $X = 1$, $Y = 0$



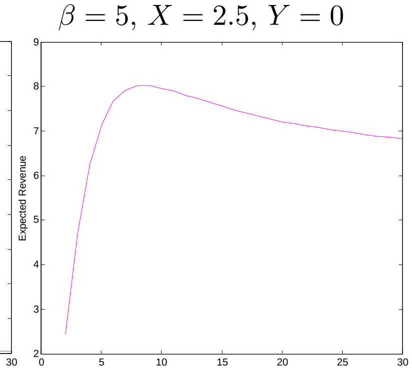
(c) Unif$[30, 60]$, $P_0 = 45$, $\beta = 5$, $X = 2.5$, $Y = 0$



(d) Unif$[30, 60]$, $P_0 = 45$, $\beta = 5$, $X = 2.5$, $Y = 0$



(e) Unif$[20, 50]$, $P_0 = 35$, $\beta = 25$, $X = 0.1$, $Y = 0$



(f) Unif$[20, 50]$, $P_0 = 35$, $\beta = 8$, $X = 1$, $Y = 0$

Figure 2: Plots of expected social welfare (a)(b)(c) and seller's revenue (d)(e)(f) against number of bidders N. One million simulations in SPA with equity bids and uniformly distributed θ. For exposition, β is specified instead of the primitives r, μ, and σ.

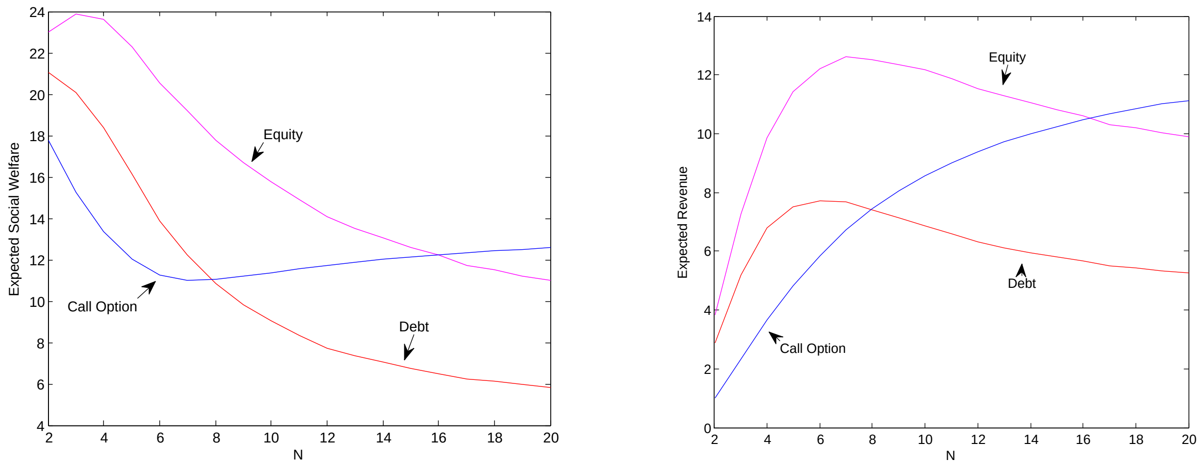
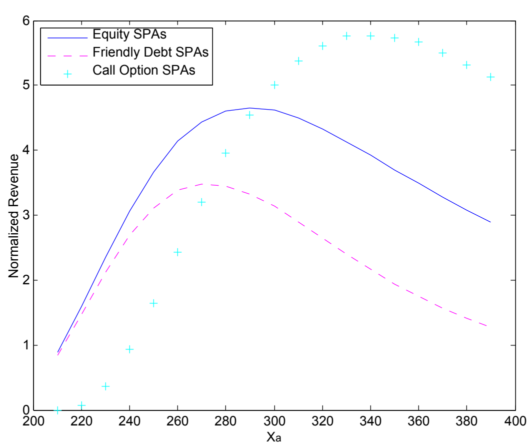
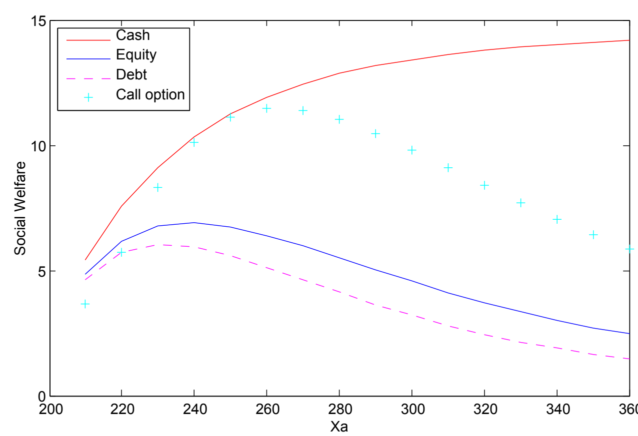


Figure 3: Plots of expected social welfare and seller's revenue against number of bidders N for SPAs with equities, friendly debts as defined in Section 4.2, and call options. One million simulations with θ uniformly distributed in $[20, 50]$, $P_0 = 35$, $r = 0.123$, $\mu = 0.001$, $\sigma = 0.05$, and $X + Y = 1$.



(a) $\beta = 5$, $X = 10$, $Y = 0$, $N = 5$



(b) $\beta = 6$, $X = 8$, $Y = 0$, $N = 30$

Figure 4: Plots of expected seller's revenue and social welfare against the auction threshold for SPAs with equities, friendly debts as defined in Section 4.2, and call options. One million simulations for θ uniformly distributed in $[200, 500]$, present value normalized at cashflow at $P = 210$. For exposition, β is specified instead of the primitives r, μ, and σ.